MIMO
What is real, what is Wishful Thinking?

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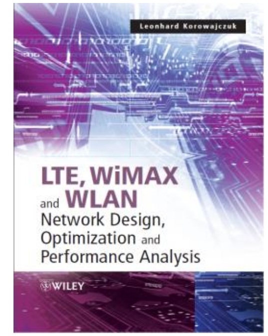
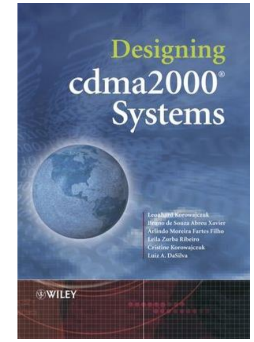
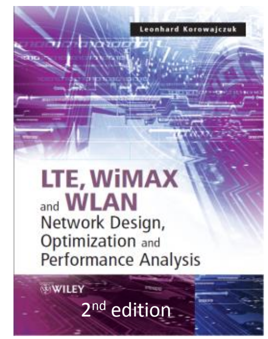
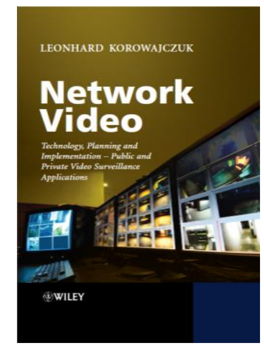
Presenter

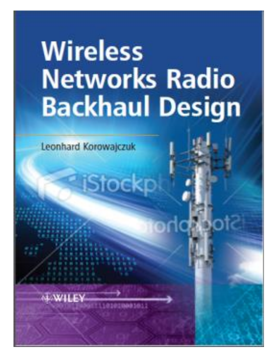
- **Leonhard Korowajczuk**
  - CEO/CTO CelPlan International
  - 45 years of experience in the telecom field (R&D, manufacturing and service areas)
  - Holds13 patents
  - Published books
    - “Designing cdma2000 Systems”
      - published by Wiley in 2006- 963 pages, available in hard cover, e-book and Kindle
    - “LTE , WiMAX and WLAN Network Design, Optimization and Performance Analysis ”
      - published by Wiley in June 2011- 750 pages, available in hard cover, e-book and Kindle
  - Books in Preparation:
    - LTE , WiMAX and WLAN Network Design, Optimization and Performance Analysis
      - second edition (2014) LTE-A and WiMAX 2.1(1,000+ pages)
    - Network Video: Private and Public Safety Applications (2014)
    - Backhaul Network Design (2015)
    - Multi-Technology Networks: from GSM to LTE (2015)
    - Smart Grids Network Design (2016)

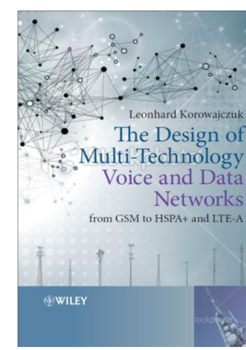












CelPlan International

- Employee owned enterprise with international presence
  - Headquarters in USA
  - 450 plus employees
  - Twenty (20) years in business
- Subsidiaries in 6 countries with worldwide operation
- Vendor Independent
- Network Design Software (CelPlanner Suite/CellDesigner)
- Network Design Services
- Network Optimization Services
- Network Performance Evaluation
- Services are provided to equipment vendors, operators and consultants
- High Level Consulting
  - RFP preparation
  - Vendor interface
  - Technical Audit
  - Business Plan Preparation
  - Specialized (Smart Grids, Aeronautical, Windmill, …)
- Network Managed Services
- 2G, 3G, 4G, 5G Technologies
- Multi-technology / Multi-band Networks
- Backhaul, Small cells, Indoor, HetNet, Wi-Fi offloading

CelPlan Webinar Series

- **How to Dimension user Traffic in 4 G networks**
  - May 7th 2014
- **How to Consider Overhead in LTE Dimensioning and what is the impact**
  - June 4th 2014
- **How to Take into Account Customer Experience when Designing a Wireless Network**
  - July 9th 2014
- **LTE Measurements what they mean and how they are used?**
  - August 6th2014
- **What LTE parameters need to be Dimensioned and Optimized? Can reuse of one be used? What is the best LTE configuration?**
  - September 3rd 2014/ September 17th, 2014
- **Spectrum Analysis for LTE Systems**
  - October 1st 2014
- **MIMO: What is real, what is Wishful Thinking?**
  - November 5th 2014
- **Send suggestions and questions to: webinar@celplan.com**

Webinar 1 (May 2014)
How to Dimension User Traffic in 4G Networks

Participants from 44 countries
Youtube views: 1144

User Traffic

1. How to Dimension User Traffic in 4G Networks
2. How to Characterize Data Traffic
3. Data Speed Considerations
4. How to calculate user traffic?
5. Bearers
6. User Applications Determination
7. User Distribution

Webinar 2 (June 2014)
How to consider overhead in LTE dimensioning
and what is the impact

Participants from 49 countries
Youtube views: 545

Overhead in LTE

1. Reuse in LTE
2. LTE Refresher
    1. Frame
    2. Frame Content
    3. Transmission Modes
    4. Frame Organization
        1. Downlink Signals
        2. Uplink Signals
        3. Downlink Channels
        4. Uplink Channels
    5. Data Scheduling and Allocation
    6. Cellular Reuse
3. Dimensioning and Planning
4. Capacity Calculator

Webinar 3 (July 2014)
How to consider Customer Experience when designing a wireless network

Participants from 40 countries

Youtube views: 467

Customer Experience

1. How to evaluate Customer Experience?
2. What factors affect customer experience?
3. Parameters that affect cutomer experience
4. SINR availability and how to calculate it
5. Conclusions
6. New Products

Webinar 4 (August 6th, 2014)
LTE Measurements
What they mean?
How are they used?

Participants from 44 countries

Youtube views: 686

LTE Measurements

1. Network Measurements
    1. UE Measurements
        - RSRP
        - RSSI and its variations
        - RSRQ and its variations
        - RSTD
        - RX-TX Time Difference
    2. Cell Measurements
        - Reference Signal TX Power
        - Received Interference Power
        - Thermal Noise Power
        - RX-TX Time Difference
        - Timing Advance
        - Angle of Arrival
    3. Measurement Reporting
        - Intra-LTE
        - Inter-RAT
        - Event triggered
        - Periodic
2. Field Measurements
    1. 1D Measurements
        - RF propagation model calibration
        - Receive Signal Strength Information
        - Reference Signal Received Power
        - Reference Signal Received Quality
        - Primary Synchronization Signal power
        - Signal power
        - Noise and Interference Power
        - Fade Mean
    2. 2D Measurements
        - Primary Synchronization Signal Power Delay Profile
    3. 3D measurements
        - Received Time Frequency Resource Elements
        - Channel Frequency response
        - Channel Impulse Response
        - Transmit Antenna Correlation
        - Traffic Load
    4. Measurement based predictions

Webinar 5 (September 3rd, 2014)

What LTE parameters need to be Dimensioned and Optimized

Part 1- Downlink

Participants from 69 countries

Youtube views: 1220

Webinar 5 (September 16th, 2014)

What LTE parameters need to be Dimensioned and Optimized

Part 2- Uplink

Participants from 46 countries

Youtube views: 316

Webinar 6
Spectrum Analysis for LTE Systems

October 1st 2014

Participants from x countries
Youtube views: 145

Spectrum Analysis for LTE Systems

- LTE is an OFDM broadband technology, with very wide channels. Narrow band channels present similar fading characteristics in its bandwidth, with variations restricted only to time dimension. Wide band channels vary in the frequency domain also. The designer has to have a full understanding of this variations and this information is not available with traditional test gear
- Until today designers had to guess multipath and fading performance, but the deployment of wide band channels and MIMO techniques require a precise understanding of this effect geographically
  - This requires 2D and 3D analysis
- Decisions as where to deploy cells, what number of antennas to use and parameter settings, can represent huge capital (CAPEX) savings and reduce operational costs (OPEX)

- RF Parameter Characterization in Broadband Channels
- Traditional Spectrum Analysis
- LTE Performance Spectrum Analysis
- Network Characterization though Drive Test
- Drive Test Devices
  - Software Defined Receivers
  - Spectrum recording
- Visualizing Measurements in Multiple Dimensions
  - 1 Dimension
  - 2 Dimensions
  - 3 Dimensions
- Measurement Interpolation and Area Prediction
- Explaining LTE Measurement Content
  - RX Signal Strength per RE
  - Noise Filtered Channel Response for each RS
  - RF Channel Response for RS carrying OFDM symbols
  - RF Channel Response for all OFDM symbols
  - Impulse Response for each RS Carrying OFDM symbol
  - Multipath Delay Spread
  - Reference Signal Received Power
  - Receive Signal Strength Indicator: full OFDM symbols
  - Receive Signal Strength Indicator: RS RE of OFDM symbols
  - Receive Signal Strength Indicator: PBCH
  - Reference Signal Received Quality: full OFDM symbols
  - Reference Signal Received Quality: RS RE of OFDM symbols
  - Reference Signal Received Quality: PBCH
  - PSS Power Distribution Profile
  - PSS Power
  - Frequency Fade Mean
  - Frequency Fade Variance
  - Signal power
  - Noise Power
  - Signal to Noise and Interference Ratio
  - Antenna Correlation
  - LTE Frame Traffic Load

LTE Technology, Network Design & Optimization Boot Camp

December 8 to 12, 2014

at University of West Indies (UWI)

St. Augustine, Trinidad

LTE Technology, Network Design & Optimization Boot Camp

- December 8 to 12, 2014
- Based on the current book and updates from the soon-to-be published 2nd edition of, "***LTE, WiMAX, and WLAN: Network Design, Optimization and Performance Analysis***", by Leonhard Korowajczuk, this -day course presents students with comprehensive information on LTE technology, projects, and deployments.
- CelPlan presents a realistic view of LTE networks, explaining what are just marketing claims and what can be achieved in real life deployments. Each module is taught by experienced 4G RF engineers who design and optimize networks around the globe.
- The materials provided are based upon this experience and by the development of industry leading planning & optimization tools, such as the CelPlanner Software Suite, which is also provided as a 30-day demo to each student
- **Module A: LTE Technology**
  - Signal Processing Applied to Wireless Communications
  - LTE Technology Overview
  - Connecting to an LTE network: an UE point of view
  - How to calculate the capacity of an LTE cell and network
  - Understanding scheduling algorithms
  - LTE measurements and what they mean
  - Understanding MIMO: Distinguishing between reality and wishful thinking
  - Analyzing 3D RF broadband drive test

LTE Technology, Network Design & Optimization Boot Camp

- **Module B: LTE Network Design**
  - Modeling the LTE Network
  - Building Network Component Libraries
  - Modeling user services and traffic
  - Creating Traffic Layers
  - RF Propagation Models and its calibration
  - Signal Level Predictions
  - LTE Predictions
  - LTE Parameters
  - LTE Resource Optimization
  - LTE Traffic Simulation
  - LTE Performance
  - Interactive Workshop (sharing experiences)
- 4G Certification (Optional)
- Additional information, Pricing & Registration available at www.celplan.com

Today's Feature Presentation

Webinar 7
MIMO
What is Real?
What is Wishful Thinking?

November 5th 2014

Content

1. Support Theory

Mathematics

Signal Processing

1.1 Mathematics

Polynomial Decomposition

Exponential Number

Matrixes

Polynomial Decomposition

- In 1712, Brook Taylor demonstrated that a differentiable function around a given point can be approximated by a polynomial whose coefficients are the derivatives of the function at that point
- The Taylor series is then a representation of a function as an infinite sum of terms calculated from its derivatives at a single point. If this series is centered at zero it is called a MacLaurin series
- A function f(x), differentiable at a point a, can be approximated by the polynomial in equation

$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f^{(3)}(a)}{3!}(x-a)^3 + \ldots\ldots.$	**Approximation of differentiable function f(x)**
$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f^{(3)}(0)}{3!}x^3 + \ldots\ldots.$	**MacLaurin series**

$sin'(x) = cos(x)$	**Differentiation of sin (x)**
$cos'(x) = -sin(x)$	**Differentiation of cos (x)**
$sin(0) = 0$	**Sin of 0**
$cos(0) = 1$	**Cos of 0**
$cos(\theta) = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!}$	**Cosine decomposition**
$\sin(\theta) = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!}$	**Sine decomposition**

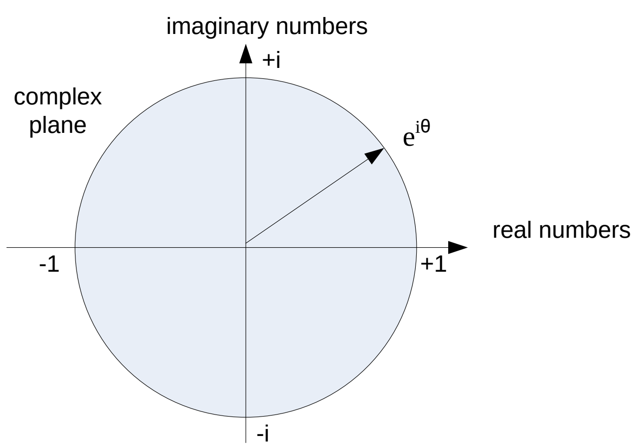
Exponential Number (e)

- The power function $f(x) = a^x$ is very useful to represent variations that are experienced in real life and its representation by polynomials has been sought
- When we apply Taylor's expansion to the exponential function we have to calculate its derivative at a point, which is given by equation

$f'(x_0) = \lim[(f(x_0+\delta) - f(x_0))/\delta]$	**Function derivative at a point when δ→0**
$f'(a^{x_0}) = lim[(a^{\delta}-1)/\delta]$	**Derivative of a^{x_0} when δ→0**

- Calculating the above limit when δ→0, we see that it is less than 1 for a =2 and greater than 1 for a=3. It is possible then to find a value of a, for which this limit is equal to 1 and the derivative of the function will then be itself. This value is irrational and is approximated by 2.718281….
- This mathematical constant is a unique real number with its derivative at a point x=0 exactly equal to itself. This number is called the exponential number and is represented by "e".
- The MacLaurin expansion of $e^{i\theta}$ is given by

$e^{i\theta} = 1 + i\theta - \frac{\theta^2}{2!} - \frac{i\theta^3}{3!} + \frac{\theta^4}{4!} + \frac{i\theta^5}{5!} - \frac{\theta^6}{6!} - \frac{i\theta^7}{7!} + \cdots$	**MacLaurin expansion of $e^{i\theta}$**
$Re\{e^{i\theta}\} = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \cdots$	**Real part of $\{e^{i\theta}\}$**
$Im\{e^{i\theta}\} = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \cdots$	**Imaginary part of $\{e^{i\theta}\}$**
$e^{i\theta} = cos(\theta) + i\, sin(\theta)$	**Euler's formula $\{e^{i\theta}\}$ as a function f sin and cos**



Matrices

Matrices

- A **matrix** is a rectangular array of numbers arranged in rows and columns
- Matrix Notation (row column) $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$
- Representation of equations

$$\begin{aligned} x + y &= 0 \\ y + z &= 3 \\ z - x &= 2 \end{aligned} \qquad \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 3 \\ -1 & 0 & 1 & 2 \end{bmatrix}$$

- Matrix size is defined by rows x columns: above matrix is 3x4 (3 by 4)
- Type of matrices
  - Regular: square or retangular

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \qquad \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

  - Upper Triangular, Lower Triangular, Diagonal, Identity

$$\begin{bmatrix} 3 & 4 & 5 \\ 0 & 2 & 2 \\ 0 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 3 & 0 & 0 \\ 4 & 2 & 0 \\ 5 & 2 & 1 \end{bmatrix} \qquad \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Matrix Operations

- Addition: $\begin{bmatrix} 0 & 1 \\ 9 & 8 \end{bmatrix} + \begin{bmatrix} 6 & 5 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 6 \\ 12 & 12 \end{bmatrix}$
  – Matrices must have same size
- Subtraction: $\begin{bmatrix} 0 & 1 \\ 9 & 8 \end{bmatrix} - \begin{bmatrix} 6 & 5 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} -6 & -4 \\ 6 & 4 \end{bmatrix}$
  - Matrices must have same size
- Scalar Product: $2 * \begin{bmatrix} 0 & 1 \\ 9 & 8 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 18 & 16 \end{bmatrix}$

Matrix Operations

- Product of A*B
  - Is done by multiplying the rows of A by columns of B
  - The product of the *i*-th row of *A* and the *j*-th column of *B* is the *i,j*-th entry of the product matrix *AB*
  - For *AB* to exist, *A* must have the same number of columns as *B* has rows
  - A*B can have a different result from B*A
    - (2x3)*(3x2)= (2x2)
    - (3x2)*(2x3)= (3x3)
    - (2x4)*(3x2)= does not exist

$$A = \begin{bmatrix} 2 & -1 \\ 0 & 3 \\ 1 & 0 \end{bmatrix} \quad B=\begin{bmatrix} 0 & 1 & 4 \\ -2 & 0 & 2 \end{bmatrix} \quad A * B = \begin{bmatrix} 2 & -1 \\ 0 & 3 \\ 1 & 0 \end{bmatrix} * \begin{bmatrix} 0 & 1 & 4 \\ -2 & 0 & 2 \end{bmatrix} \quad (3x2) * (2x3) = (3x3)$$

$$A * B = \begin{bmatrix} 2*0+(-1)*(-2) & 2*1+(-1)*0 & 2*4+(-1)*2 \\ 0*0+3*(-2) & 0*1+3*0 & 0*4+3*2 \\ 1*0+0*(-2) & 1*1+1*0 & 1*4+0*2 \end{bmatrix}$$

$$A * B = \begin{bmatrix} 2 & 2 & 6 \\ -6 & 0 & 6 \\ 0 & 1 & 4 \end{bmatrix}$$

Matrix Operations

- Matrix row operations: this operations do not affect the matrix
  - Row switching $\begin{bmatrix}0 & 0 & 1\\ 1 & 2 & 3\\ 0 & 1 & -2\end{bmatrix} \rightarrow \begin{bmatrix}1 & 2 & 3\\ 0 & 1 & -2\\ 0 & 0 & 1\end{bmatrix}$
  - Row multiplication $\begin{bmatrix}0 & 2 & 4\\ 1 & 2 & -3\\ 0 & -2 & -5\end{bmatrix} \xrightarrow[\text{-R3}]{\text{½ R1}} \begin{bmatrix}0 & 1 & 2\\ 1 & 2 & -3\\ 0 & 2 & 5\end{bmatrix}$
  - Row Adition $x + 2y = 1 \quad -2x + 3y = 5 \quad \begin{bmatrix}1 & 2 & 1\\ -2 & 3 & 5\end{bmatrix}$

$$\begin{bmatrix}1 & 2 & 1\\ -2 & 3 & 5\end{bmatrix} \xrightarrow{\text{2R1+R2}} \begin{bmatrix}1 & 2 & 1\\ 0 & 7 & 7\end{bmatrix}$$

Matrix Reduction

- Echelon form of a matrix
  - A matrix is in **row echelon form** (ref) when it satisfies the following conditions
    - The first non-zero element in each row, called the **leading entry**, is 1
    - Each leading entry is in a column to the right of the leading entry in the previous row
    - Rows with all zero elements, if any, are below rows having a non-zero element

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

- A matrix is in **reduced row echelon form** (rref) when it satisfies the following conditions.
  - The matrix satisfies conditions for a row echelon form
  - The leading entry in each row is the only non-zero entry in its column

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

Matrix Rank

- Matrix Rank
  - The rank of a matrix is defined as
    - The maximum number of linearly independent *column* vectors in the matrix or
    - The maximum number of linearly independent *row* vectors in the matrix. Both definitions are equivalent. For an r x c matrix:
      - If *r* is less than *c*, then the maximum rank of the matrix is *r*
      - If *r* is greater than *c*, then the maximum rank of the matrix is *c*
  - The maximum number of linearly independent vectors in a matrix is equal to the number of non-zero rows in its row echelon matrix
    - Therefore, to find the rank of a matrix, we simply transform the matrix to its row echelon form and count the number of non-zero rows

Matrix Determinant

- The **determinant** is a unique number associated with a square matrix $|A| = Det\ A$
- Determinant of a 2 x 2 Matrix
  - $|A| = (a_{11} * a_{22}) - (a_{12} * a_{21})$
- Determinant of a 3 x 3 Matrix
  - Rule of Sarrus: the sum of the products of three diagonal north-west to south-east lines of matrix elements, minus the sum of the products of three diagonal south-west to north-east lines of elements

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{11} & a_{12} \\ a_{21} & a_{22} & a_{23} & a_{21} & a_{22} \\ a_{31} & a_{32} & a_{33} & a_{31} & a_{32} \end{vmatrix}$$

- Leibniz Formula

$$\det(A) = \sum_{\sigma \in S_n} sgn(\sigma) \prod_{i=1}^{n} a_i . \sigma_i$$

n= size of the square matrix

σ=all permutations of n

$sgn(\sigma)$= signature of σ (defined by even (+1) or odd number of permutation (-1))

Matrix Inversion

- The inverse of matrix **A** is another *n* x *n* matrix, denoted $\mathbf{A}^{-1}$, that satisfies the following condition:

$$AA^{-1} = A^{-1}A = I_n$$

  I_n= identity matrix

- Matrix inversion
  - There is no matrix division, but there is the concept of matrix inversion
  - The rank of a matrix is a unique number associated with a square matrix
    - If the rank of an *n* x *n* matrix is less than *n*, the matrix does not have an inverse
  - The determinant is another unique number associated with a square matrix
    - When the determinant for a square matrix is equal to zero, the inverse for that matrix does not exist

Matrix Inversion

- The matrix inversion property can be used to create the inverse

$$\begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$$

- Matrix row operations are performed to transform the right hand side into an identity matrix

$$\begin{bmatrix} 1 & 3 & 3 & 1 & 0 & 0 \\ 1 & 4 & 3 & 0 & 1 & 0 \\ 1 & 3 & 4 & 0 & 0 & 1 \end{bmatrix}$$

- Matrix row operations are performed to transform the right hand side into an identity matrix

$$\begin{bmatrix} 1 & 0 & 0 & 7 & -3 & -3 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{bmatrix}$$

- The inverted matrix is then:

$$\begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

Matrix Theorems

- Matrix
  - **A**, **B**, and **C** are matrices
  - **A'** is the transpose of matrix **A**.
  - $\mathbf{A}^{-1}$ is the inverse of matrix **A**.
  - **I** is the identity matrix.
  - *x* is a real or complex number
- Matrix Addition and Matrix Multiplication
  - $\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$ (Commutative law of addition)
  - $\mathbf{A} + \mathbf{B} + \mathbf{C} = \mathbf{A} + (\mathbf{B} + \mathbf{C}) = (\mathbf{A} + \mathbf{B}) + \mathbf{C}$ (Associative law of addition)
  - $\mathbf{ABC} = \mathbf{A}(\mathbf{BC}) = (\mathbf{AB})\mathbf{C}$ (Associative law of multiplication)
  - $\mathbf{A}(\mathbf{B} + \mathbf{C}) = \mathbf{AB} + \mathbf{AC}$ (Distributive law of matrix algebra)
  - $x(\mathbf{A} + \mathbf{B}) = x\mathbf{A} + x\mathbf{B}$
- Transposition Rules
  - $(\mathbf{A}')' = \mathbf{A}$
  - $(\mathbf{A} + \mathbf{B})' = \mathbf{A}' + \mathbf{B}'$
  - $(\mathbf{AB})' = \mathbf{B}'\mathbf{A}'$
  - $(\mathbf{ABC})' = \mathbf{C}'\mathbf{B}'\mathbf{A}'$
- Inverse Rules
  - $\mathbf{AI} = \mathbf{IA} = \mathbf{A}$
  - $\mathbf{AA}^{-1} = \mathbf{A}^{-1}\mathbf{A} = \mathbf{I}$
  - $(\mathbf{A}^{-1})^{-1} = \mathbf{A}$
  - $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$
  - $(\mathbf{ABC})^{-1} = \mathbf{C}^{-1}\mathbf{B}^{-1}\mathbf{A}^{-1}$
  - $(\mathbf{A}')^{-1} = (\mathbf{A}^{-1})'$

Other Matrix Operations

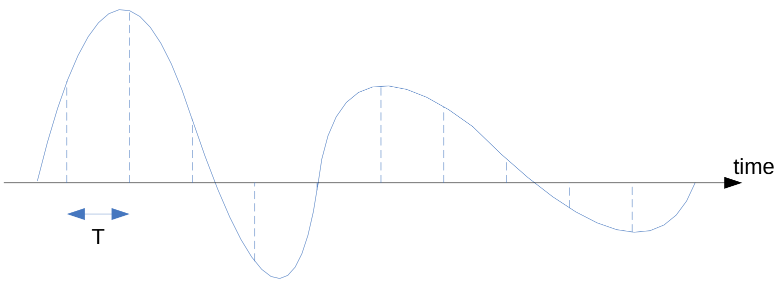
- Sum vector
- Mean vector
- Deviation scores
- Sum of Squares
- Cross Product
- Variance
- Covariance
- Norm
- Singular value

1.2 Signal Processing Fundamentals

Digitizing Analog Signals
Orthogonal Signals
Combining Sinewaves
Carrier Modulation

Digitizing analog signals

- The Sampling Theorem states that if a bandwidth limited continuous signal is sampled at a rate twice its bandwidth B it is possible to reconstruct the original signal from these samples

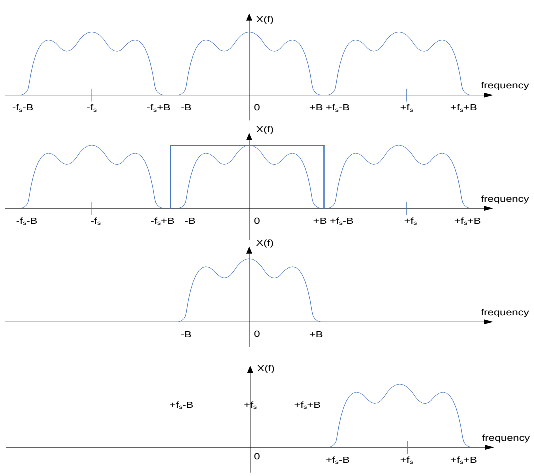


values
+3.0
+5.5
+2.0
-2.5
-0.1
+3.0
+2.5
+0.7
-0.8
-1.5
+0.0

$f_s = 2\ B$	**Nyquist sampling frequency**
$T_s = 1/f_s$	**Nyquist sampling period**

B = Maximum bandwidth frequency
f_s= Nyquist sampling frequency
T_s= Nyquist sampling period

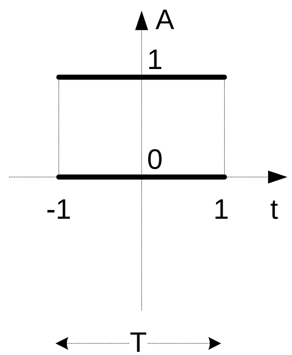
- Sampling a function x (t) creates a spectrum with a periodic function X (f)
- This spectrum has a base spectrum and images of it spaced by f_s (the sampling frequency)
  - Those images are alias of the base spectrum
  - Any of the aliases can be filtered and still convey all the required information

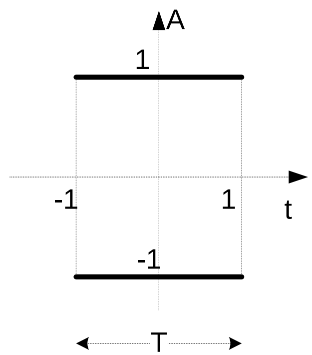


$\frac{2\ f_c - B}{m} \geq f_s \geq \frac{2\ f_c + B}{m+1}$	**Sampling frequency range**

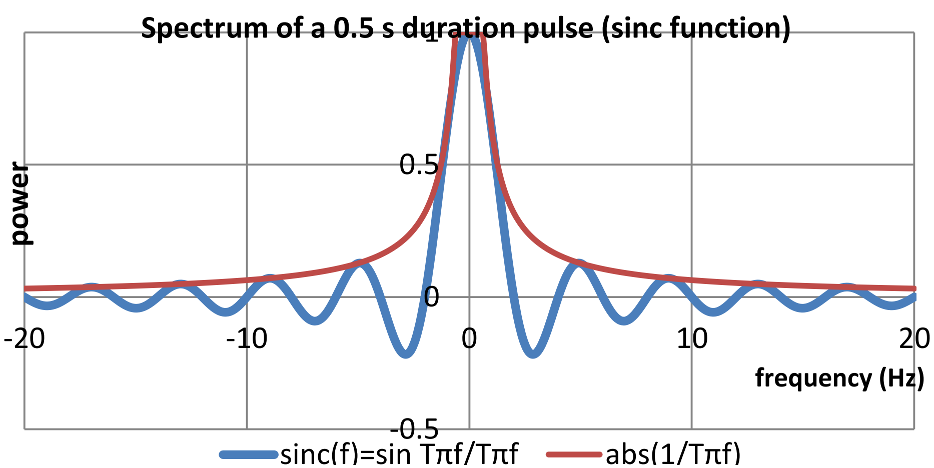
Digital data representation in the frequency domain (spectrum)

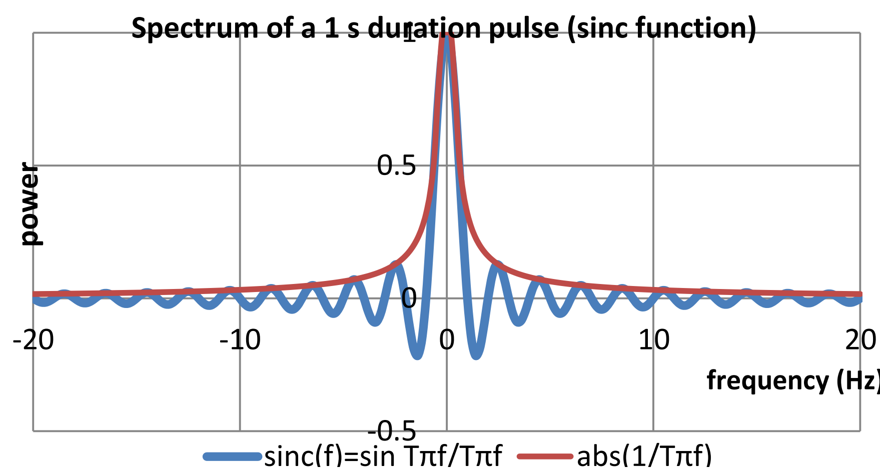
- We will start analyzing a single unit of information that can represent a value of 1 or zero and has duration defined by T (bit)
- First we will convert the bit a Non-Return to Zero (NRZ) format to eliminate the DC component as represented below
  - The bit is centered at the origin
- The Discrete Fourier Transform of this signal results in the Sinc (Sinus Cardinalis) function
- The Sinc function is equivalent to the sin (x)/x function, but the value for x=0 is pre-defined as 1





$sinc(Tf) = \frac{\sin(T\pi f)}{T\pi f}$	**Sinc function**
$B = \frac{1}{T}$	**Pulse Bandwidth**





Orthogonal Signals

- An important property of signals is their orthogonality, meaning that they can be detected independently of each other
- Two signals are considered orthogonal if their product over an entire period (dot product) is null
  - A dot product is the result of the integration of the regular product of two functions or its samples, taken over an integer number of periods
- Orthogonality also holds when signals are represented by its samples, and this property is used by the DSPs that process digital signals.
- Orthogonal signals (or their samples) can be added and the combined signal can be verified for correlation with known signals
- An auto-correlation is achieved when the combined signal is multiplied and integrated against a known signal and the result is a value different from zero
- A low cross-correlation is achieved when a known signal is not present in the combined signal, resulting in zero integration

Orthogonal Signals

- **Sine and Cosine orthogonality**
  – A sine and cosine are orthogonal to each other, as demonstrated in equations below
  – We first multiply both functions and then we integrate the resulting curve, obtaining a sum of zero
    - This can be done by multiplying the sine wave samples and adding the result for an integer number of periods

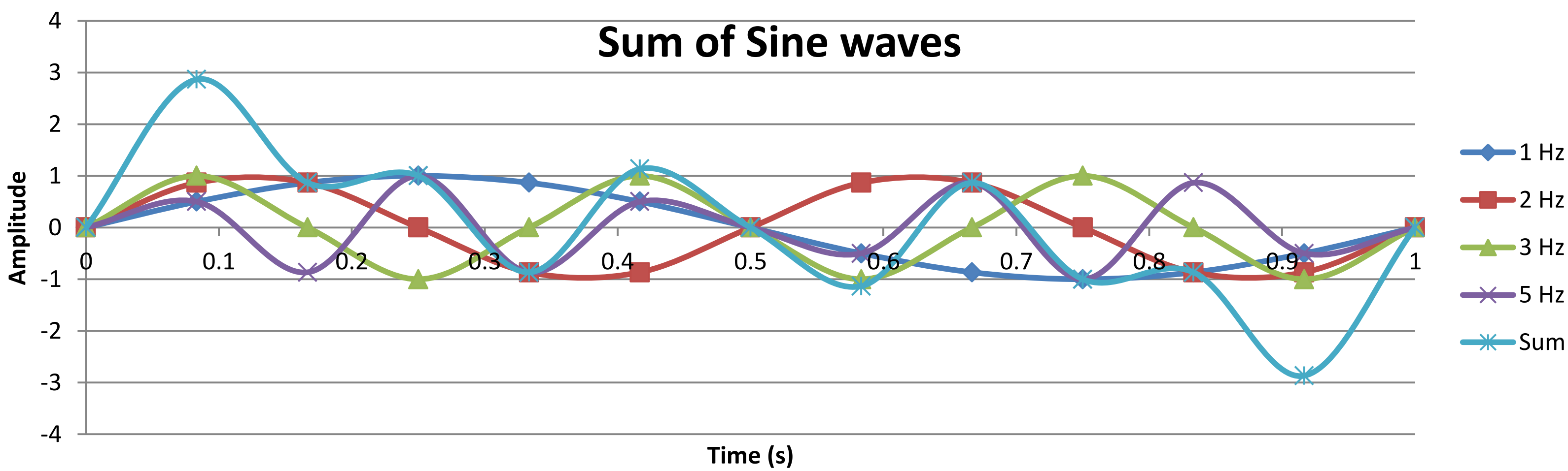
$sin\ x\ .\ cos\ x = \frac{1}{2}\sin 2x$	**Product of a sine by a cosine**
$\int_0^{2\pi} \frac{1}{2}\sin 2x = 0$	**Integral of the product of a sine by a cosine**

Orthogonal Signals

- Harmonically related signals orthogonality
  - Harmonically related signals are the ones that are multiple of each other in the frequency domain

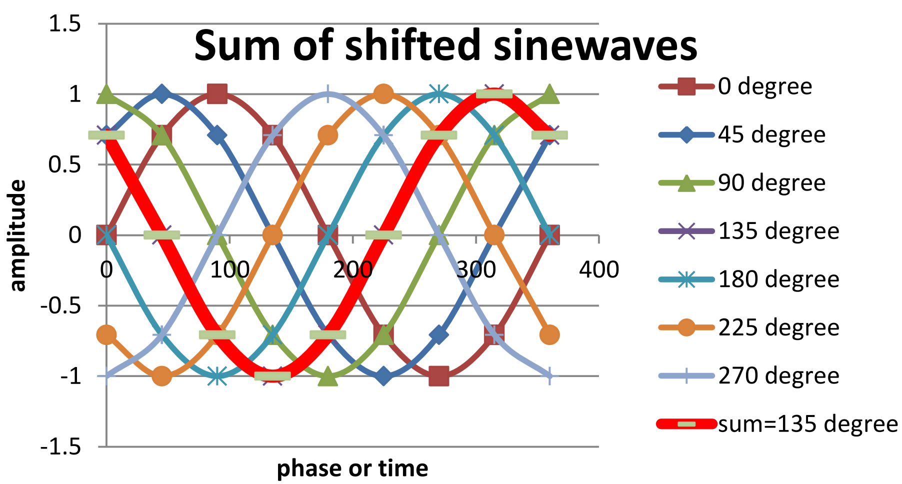
$\int_0^{2\pi} \sin x . \sin nx = 0$	**Harmonically related signal orthogonality**

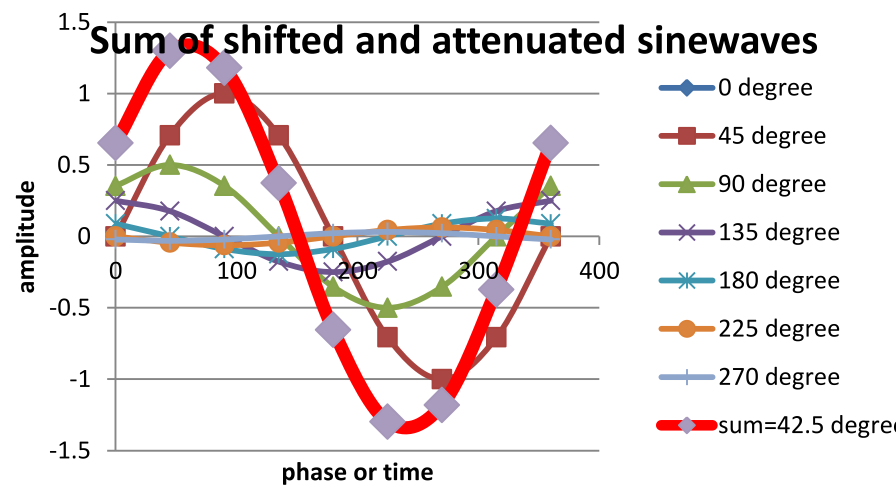
time (s)	alpha	radians	sin f1	sin f2	sin f3	sin f5	sum	sum* sin f1	sum* sin f2	sum* sin f3	sum* sin f5	sum* sin f4
0	0	0	0	0	0	0	0	0	0	0	0	0
0.083	30	0.524	0.500	0.866	1.000	0.500	2.866	1.433	2.482	2.866	1.433	0.000
0.167	60	1.047	0.866	0.866	0.000	-0.866	0.866	0.750	0.750	0.000	-0.750	0.000
0.250	90	1.571	1.000	0.000	-1.000	1.000	1.000	1.000	0.000	-1.000	1.000	0.000
0.333	120	2.094	0.866	-0.866	0.000	-0.866	-0.866	-0.750	0.750	0.000	0.750	0.000
0.417	150	2.618	0.500	-0.866	1.000	0.500	1.134	0.567	-0.982	1.134	0.567	0.000
0.500	180	3.142	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.583	210	3.665	-0.500	0.866	-1.000	-0.500	-1.134	0.567	-0.982	1.134	0.567	0.000
0.667	240	4.189	-0.866	0.866	0.000	0.866	0.866	-0.750	0.750	0.000	0.750	0.000
0.750	270	4.712	-1.000	0.000	1.000	-1.000	-1.000	1.000	0.000	-1.000	1.000	0.000
0.833	300	5.236	-0.866	-0.866	0.000	0.866	-0.866	0.750	0.750	0.000	-0.750	0.000
0.917	330	5.760	-0.500	-0.866	-1.000	-0.500	-2.866	1.433	2.482	2.866	1.433	0.000
							sum	6	6	6	6	0



Combining shifted copies of a sinewave

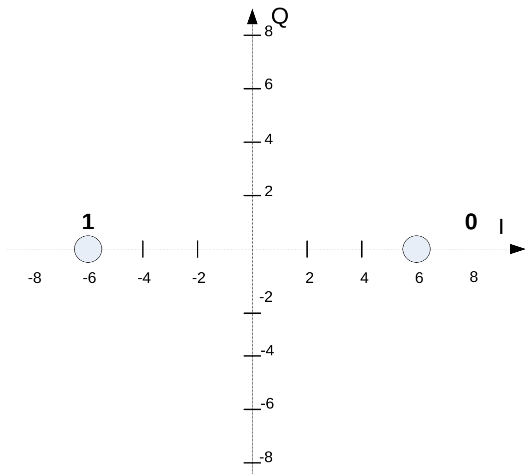
- Combined non-orthogonal signals, such as phase-shifted copies of a sinusoidal waveform result in a sinusoidal waveform that is phase shifted itself
  - The final phase shift is the average of the individual components phase shifts

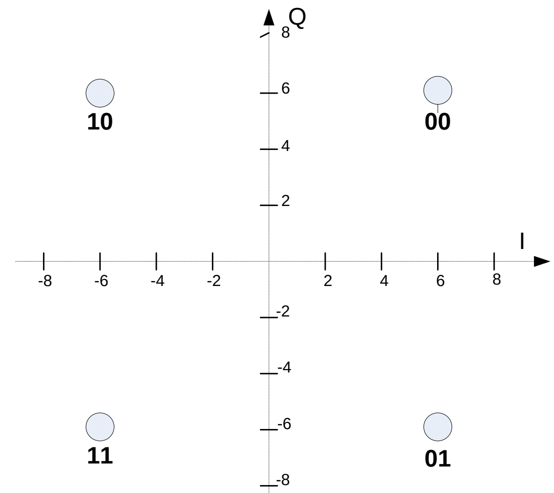


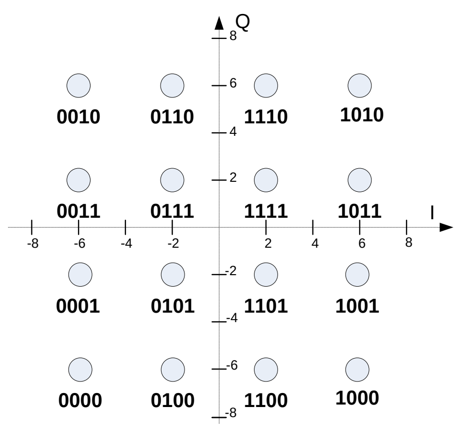


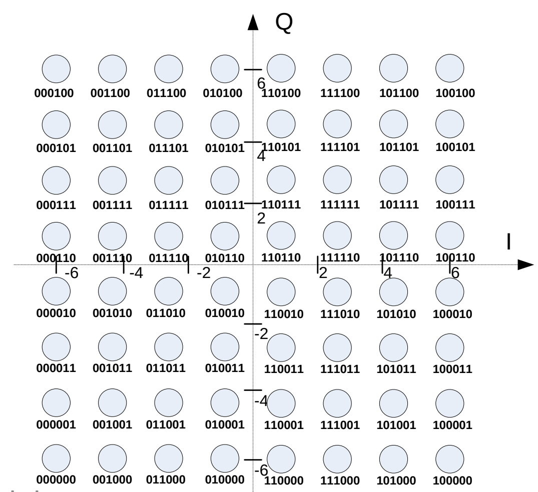
Carrier modulation

- The process of loading digital information on a carrier is called modulation
- In the modulation process, sets of bits are combined into symbols and assigned to carrier states (phase x energy) forming a constellation
- These constellations can be represented in polar form, showing the phase and magnitude in the same diagram
- The distance between constellation points represents how much noise the modulation can accommodate



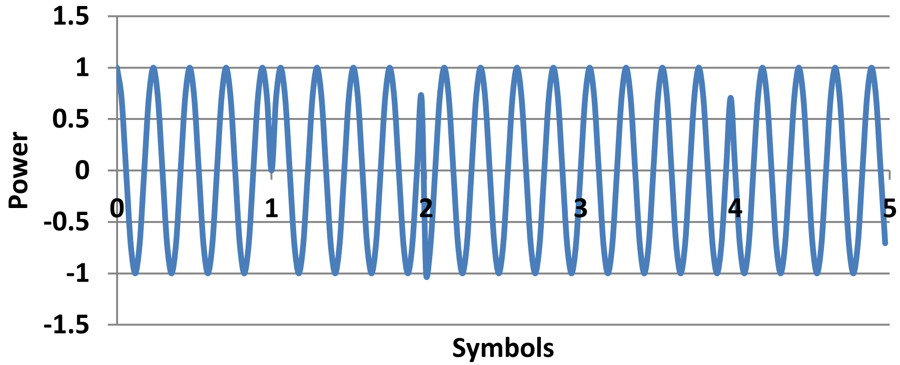




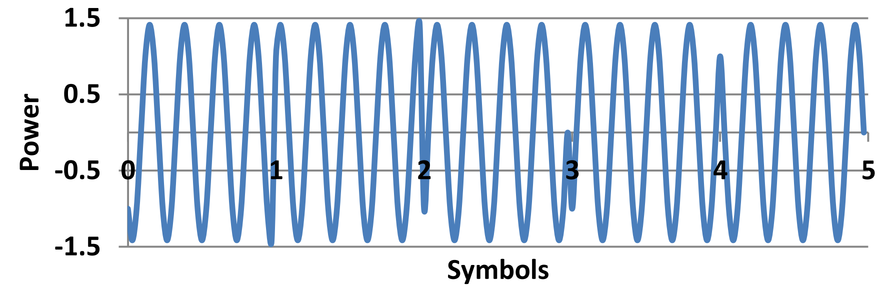


Carrier modulation

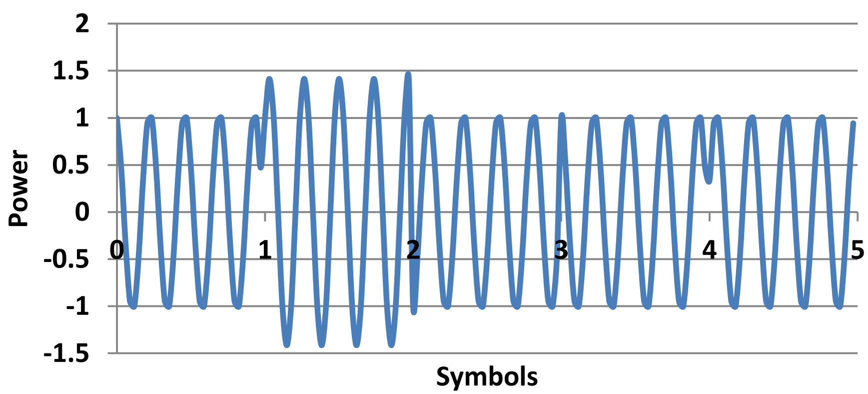
BPSK modulation (Icos-Qsin) of data bits 10110



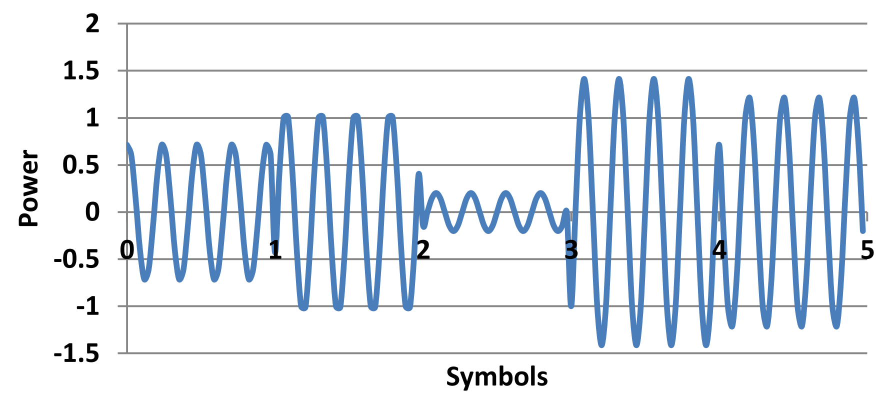
QPSK modulation of data bits 1011000110



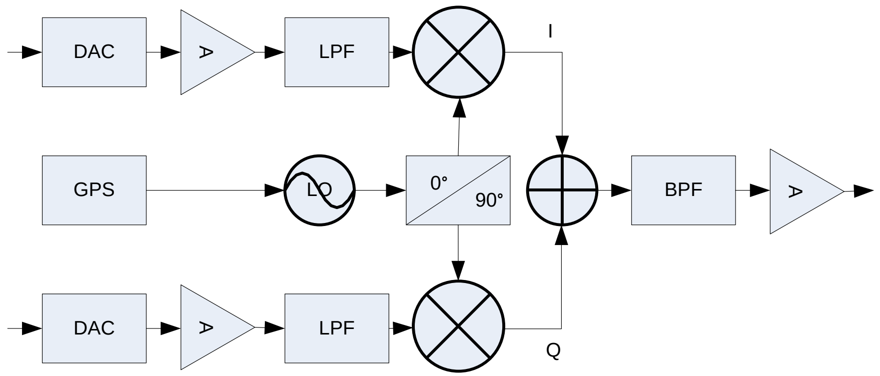
16QAM modulation of data bits
10110000101101101011



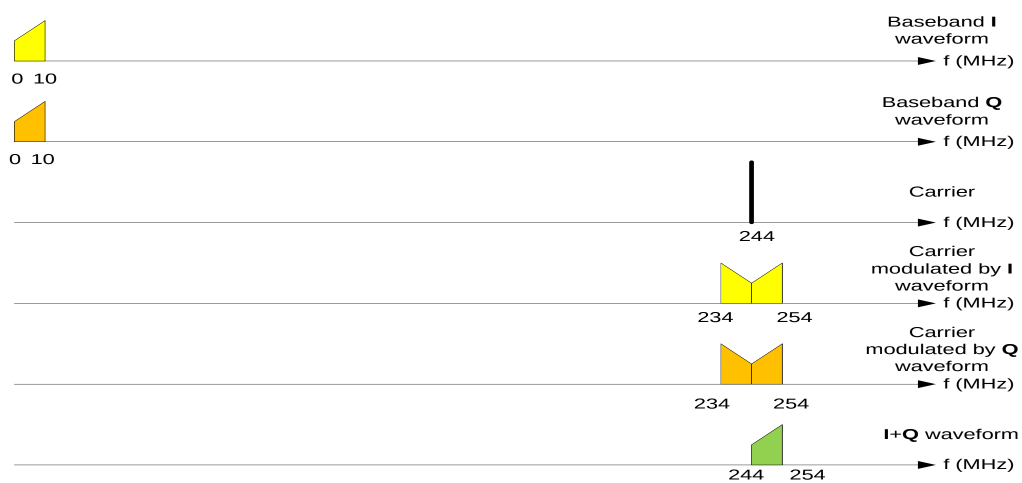
64QAM modulation of data bits
101010000111110110100000010101



Carrier modulation



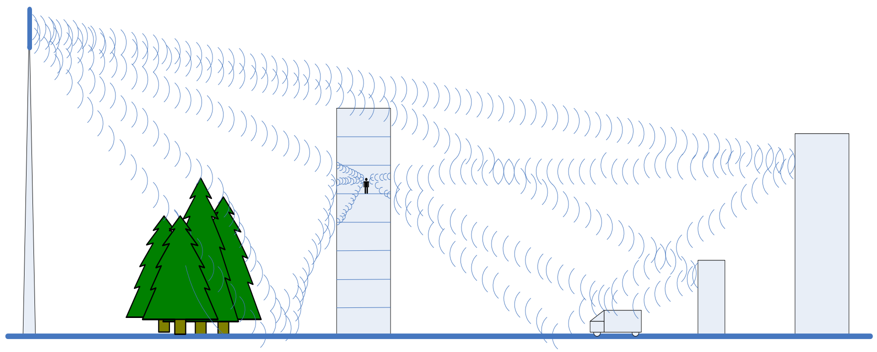
$\sin(\omega_c)\cos(\omega_m) = \frac{1}{2}\left(\sin(\omega_c + \omega_m) + \sin(\omega_c - \omega_m)\right)$	**I modulated carrier**
$\cos(\omega_c)\sin(\omega_m) = \frac{1}{2}\left(\sin(\omega_c + \omega_m) - \sin(\omega_c - \omega_m)\right)$	**Q modulated carrier**
$\sin(\omega_c)\cos(\omega_m) + \cos(\omega_c)\sin(\omega_m) = \sin(\omega_c + \omega_m)$	**I+Q modulated carrier**

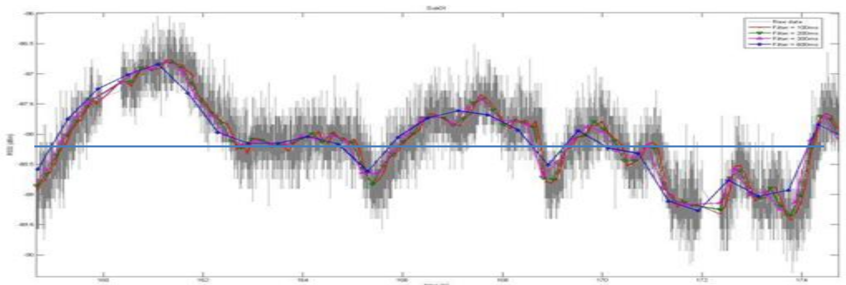


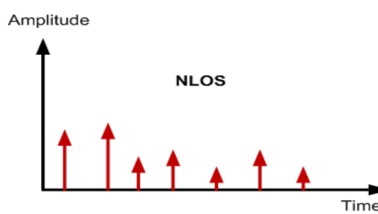
2. RF Channel

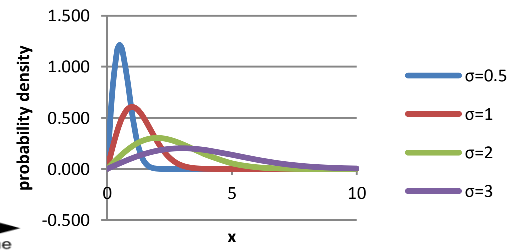
Multipath

- RF Path Characterization
  - RF Multipath and Doppler effect are the main impairments on signal propagation
  - It is essential to characterize multipath along the wireless deployment area
  - This knowledge allows the proper wireless network design
- Multipath creates fading in time and frequency
  - Broader the channel larger is the impact
  - A k factor prediction was developed to indicate the expected amount of multipath
  - This was ok for narrow band channels
- More accurate analysis is required for Broadband channels, like LTE





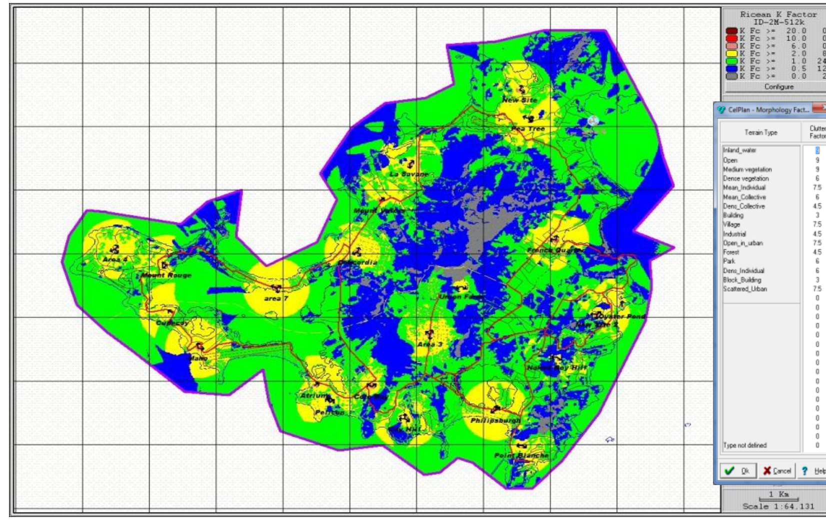




K Factor

K factor prediction

- Signals have high coherence- High k factor: >10
  - Multipath signal follows a Gaussian distribution
- Signals have medium coherence- Medium k factor: 10<k>2
  - Multipath signal follows a Ricean distribution
- Signals have low coherence- Low k factor: k<2
  - Multipath signal follows a Rayleigh distribution
- The k factor can be estimated on a pixel basis from the geographical data (topography and morphology) by RF prediction tools

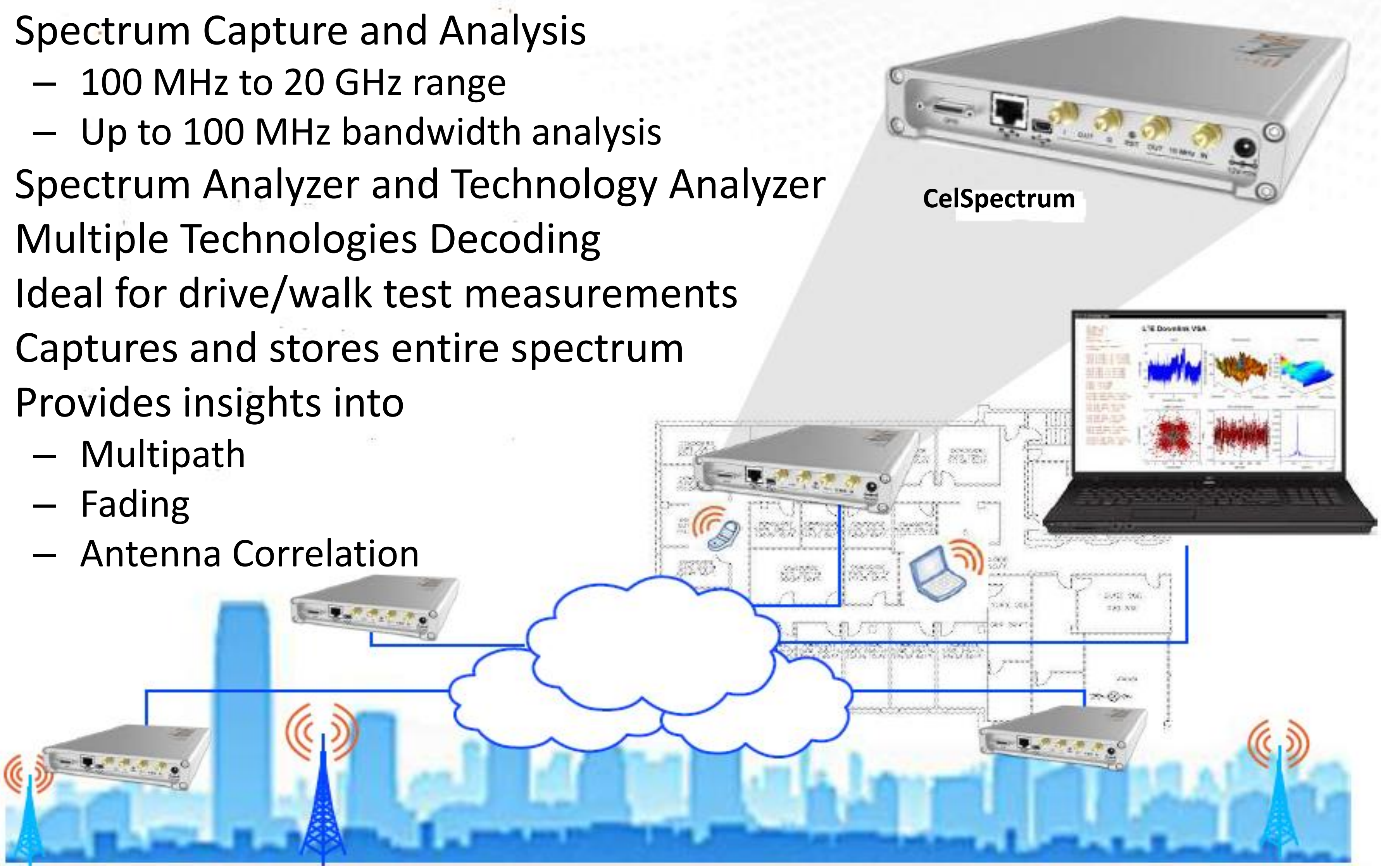


$k = 10F_s F_h F_b d^{-0.5}$	**k factor for LOS**
$F_h h_r^{/3)0.46}$	**Receive antenna height factor**
$F_b = (\frac{b}{17})^{-0.62}$	**Beam width factor**
$k = 0$	**k factor for NLOS**

F_s =Morphology factor (1 to 5), lower values apply to multipath rich environments
F_h= Receive antenna height factor
F_b=Beam width factor
d = Distance
h_r= Receive antenna height
b= Beam width in degrees

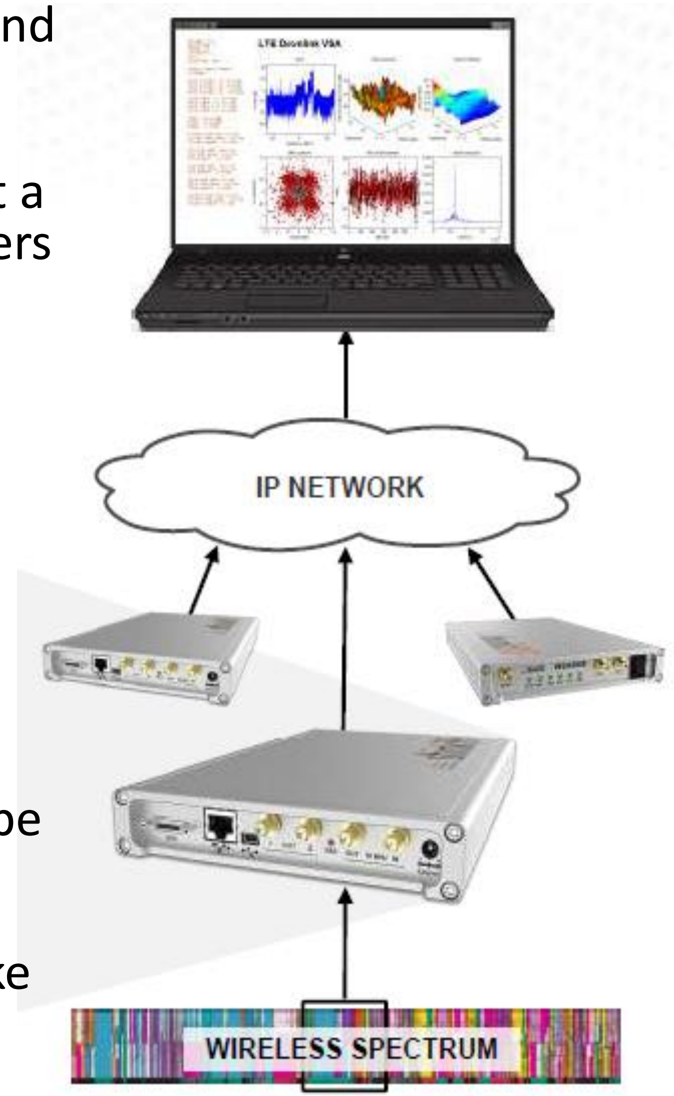
CelSpectrum™

- Universal Software Defined Receiver
- Spectrum Capture and Analysis
  - 100 MHz to 20 GHz range
  - Up to 100 MHz bandwidth analysis
- Spectrum Analyzer and Technology Analyzer
- Multiple Technologies Decoding
- Ideal for drive/walk test measurements
- Captures and stores entire spectrum
- Provides insights into
  - Multipath
  - Fading
  - Antenna Correlation



CellSpectrum™

- It is an RF Spectrum and Technology RF Channel Analyzer based on a universal software-defined receiver that enables capturing, digitizing, storing and analyzing detailed RF & technology characteristics needed for the proper design of wireless networks
- It digitizes and stores up to 100 MHz of spectrum at a time, from 100 MHz to 18 GHz, extracting parameters as:
  - LTE channel response per Resource Element
  - Multipath delay spread
  - Average frequency fading
  - Average time fading
  - Noise floor
  - Interference
  - Traffic Distribution
  - 3D visualization capability
- Additionally, allocation and traffic information can be derived, providing valuable information about the allocation used for Inter Cell Interference Coordination (ICIC). Framed OFDM transmitters, like WiMAX and LTE, provide ideal platforms to characterize the RF channel

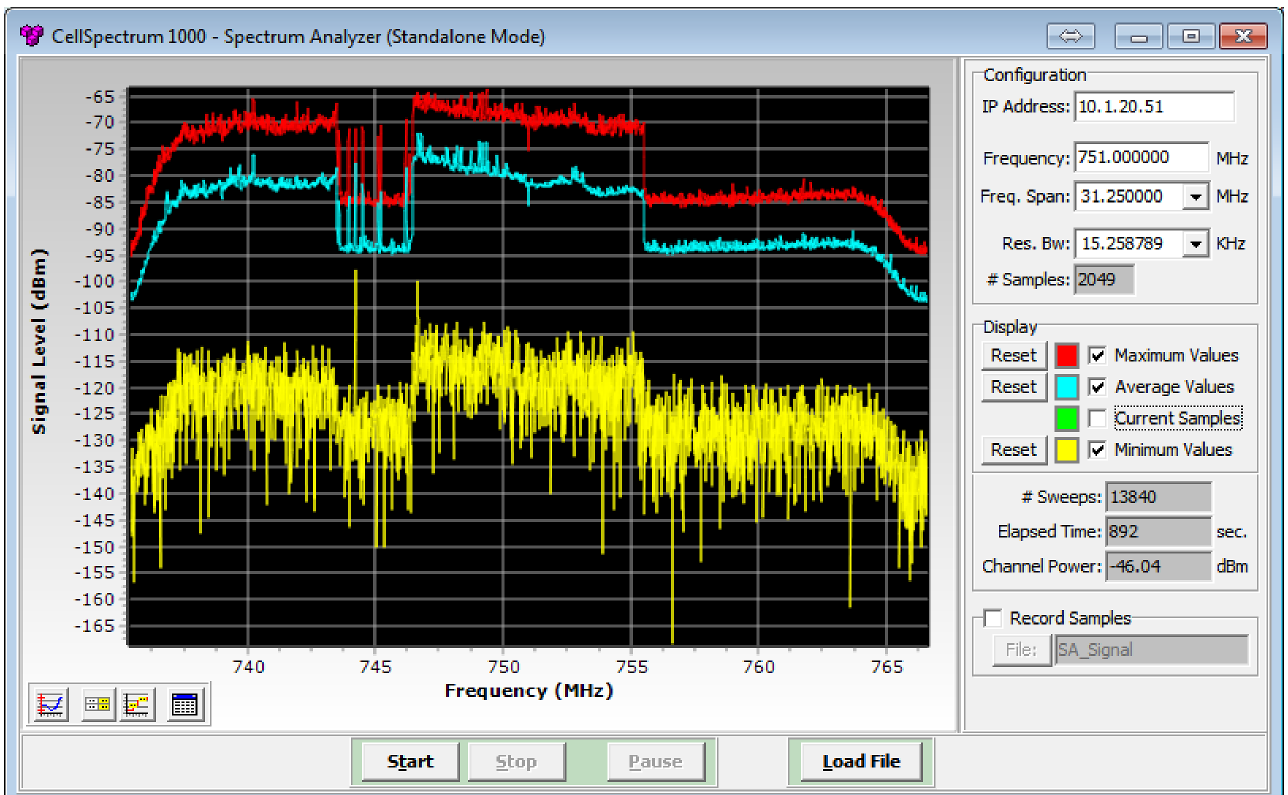


CellSpectrum

Spectrum Analyzer

Spectrum Analyzer (Stand Alone)

- User can capture live data or load data from a file
- User can use automatic display signal level range or define it manually
- Spectrum samples can be recorded in a file with a sampling rate of approximately 1 second.
- The entire displayed bandwidth is calculated, as well the power within two markers defined by the user
- Measured power can be recorded to a file
- User can access the hardware remotely



CellSpectrum (Drive Test)

- CellSpectrum allows to capture the spectrum along a drive test route and store it for future analysis
- This reduces the cost of collecting data and allows the analysis and re-analysis of it
- User can zoom to analyze part of the spectrum, as required

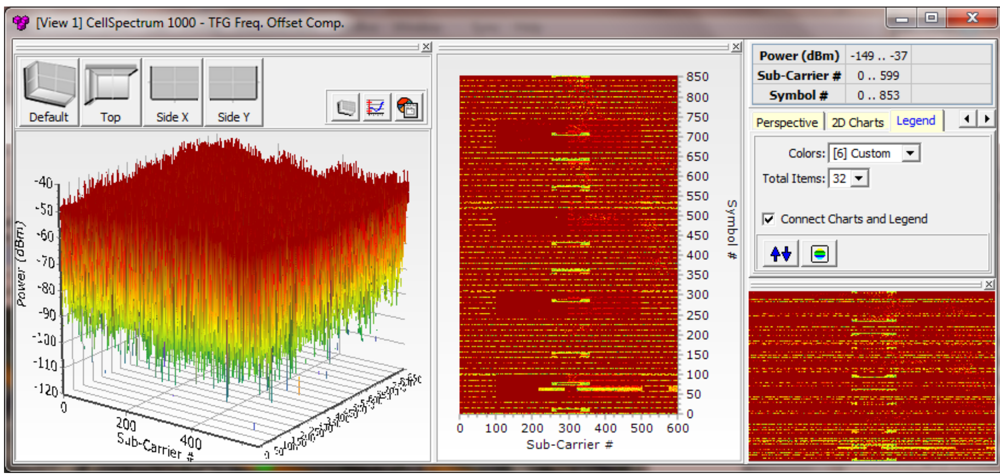
RF Channel and Technology Analyzer

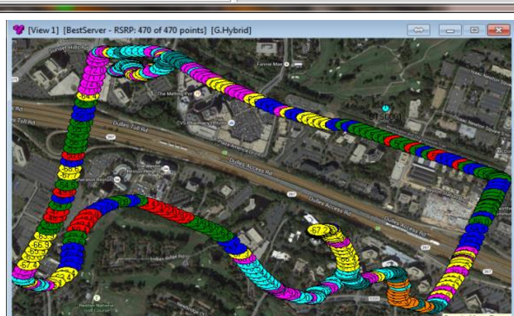
Mobile Channel characterization

Stationary Channel characterization

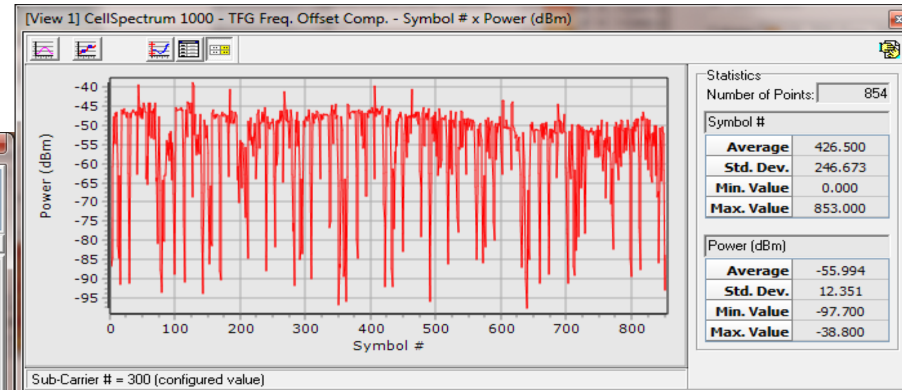
LTE Received Frame (Mobile)

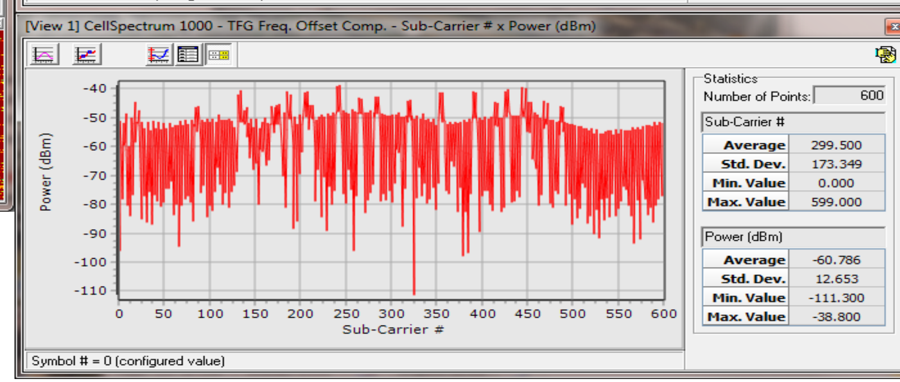
- CellSpectrum capture
- Received LOS signal after time and frequency synchronization
  - 10 MHz (600 sub-carriers)
  - 6 frames (840 symbols): 60 ms
- Signal Level per Resource Element





- Time fading

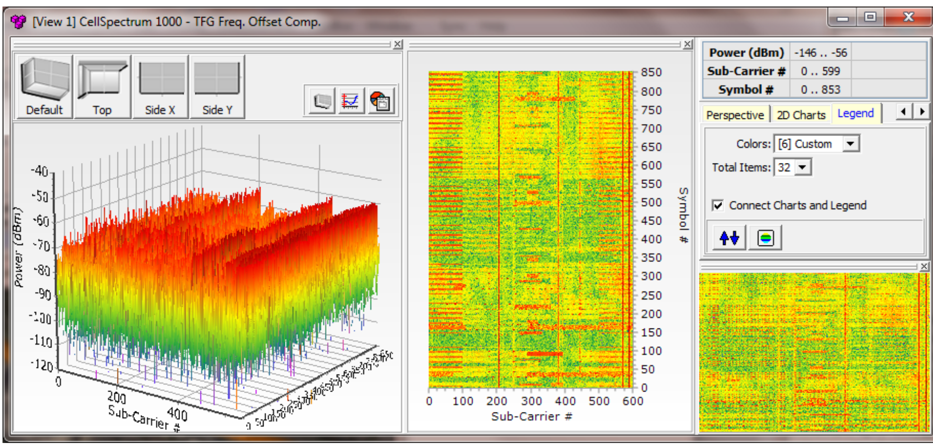




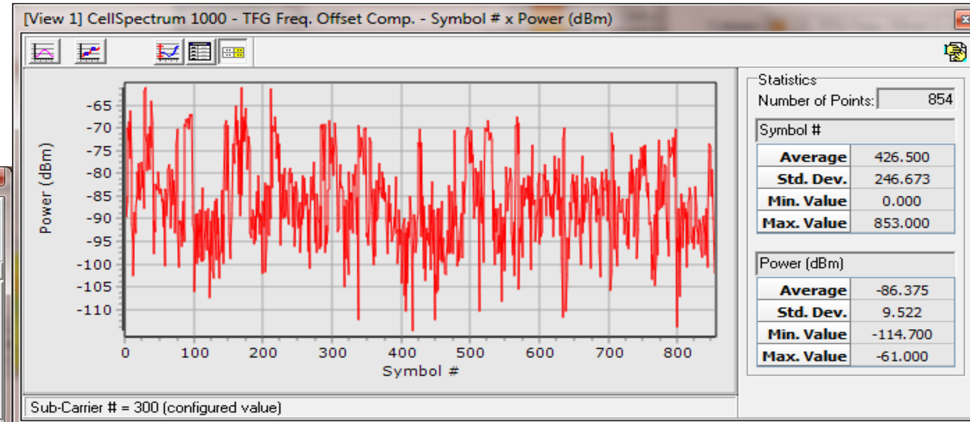
- Frequency fading

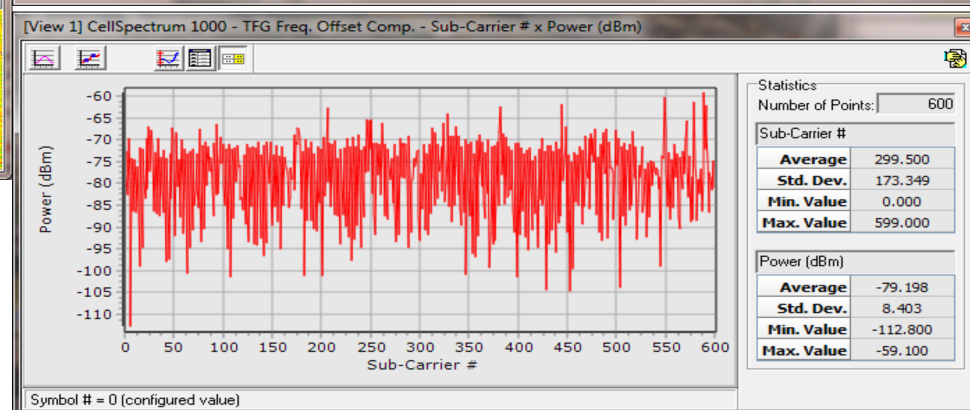
LTE Received Frame (Mobile)

- CellSpectrum capture
- Received NLOS signal after time and frequency synchronization
  - 10 MHz (600 sub-carriers)
  - 6 frames (840 symbols): 60 ms
- Signal Level per Resource Element



- Time fading

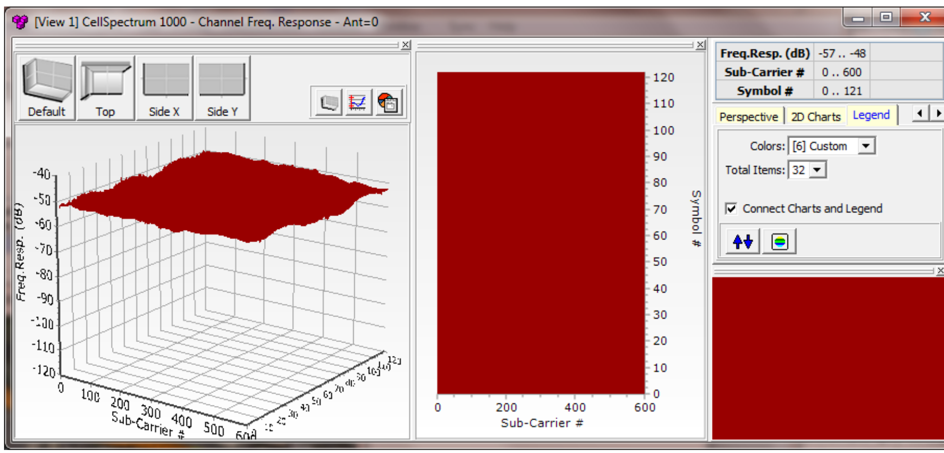




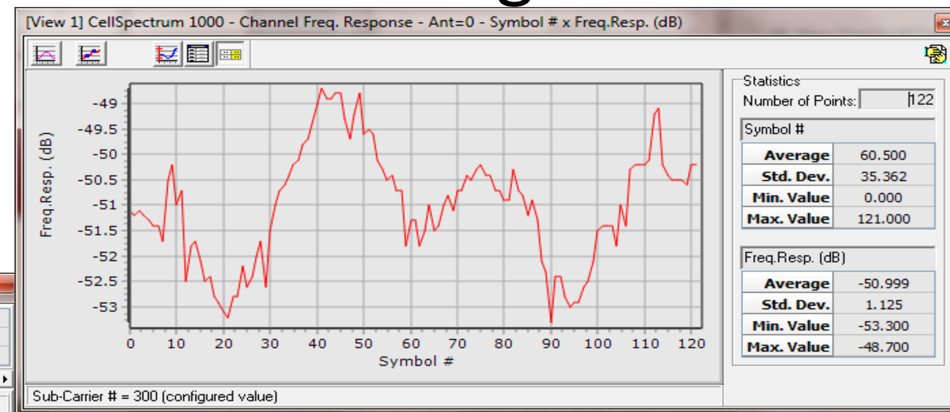
- Frequency fading

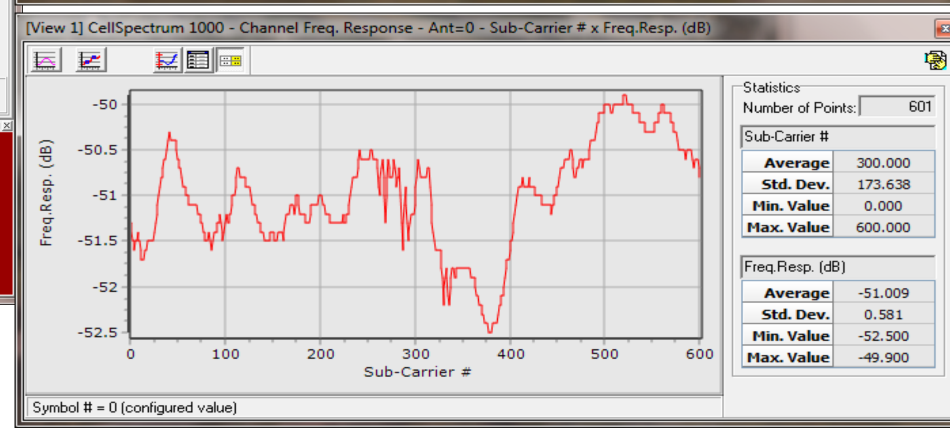
LTE RF channel LOS

- CellSpectrum capture
- Received LOS signal after time and frequency synchronization and channel equalization using pilot information
  - 10 MHz (600 sub-carriers)
  - 6 frames (840 symbols): 60 ms
- Signal Level per Resource Element



- Time fading

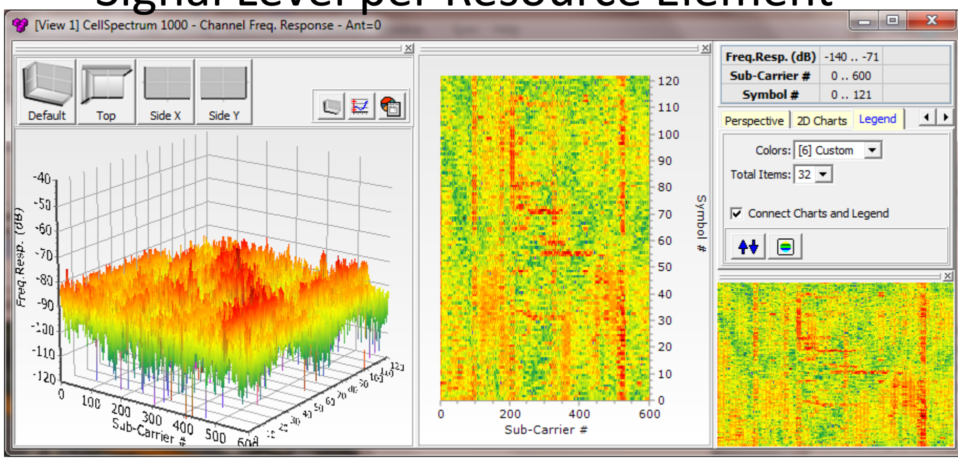




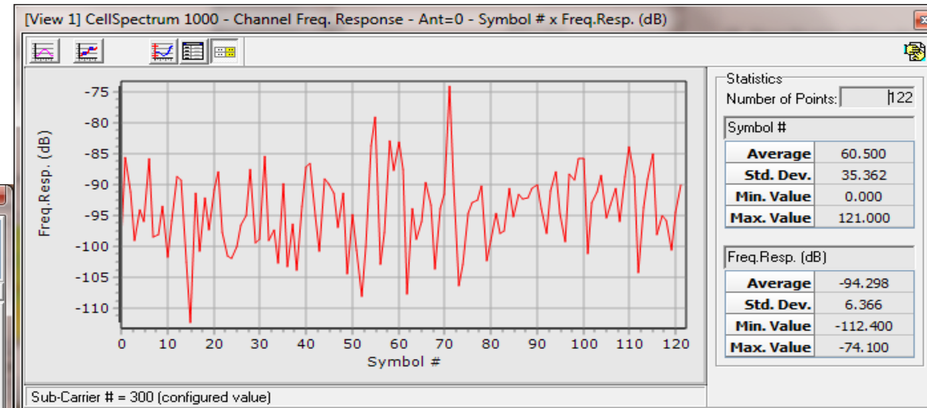
- Frequency fading

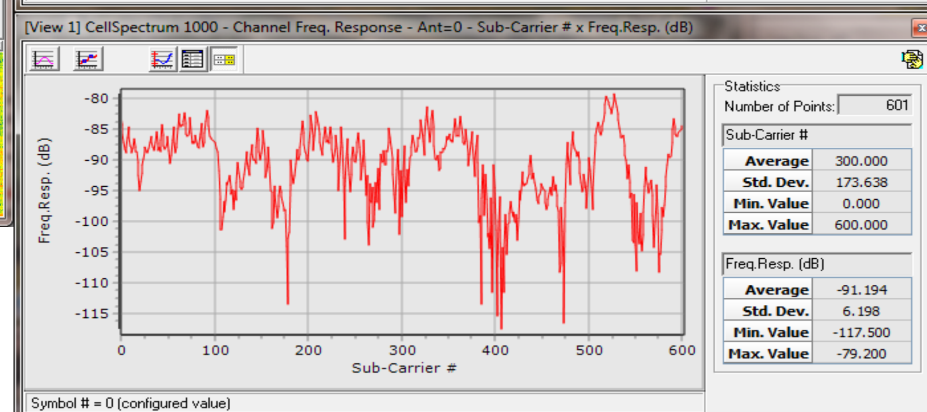
LTE RF Channel NLOS

- CellSpectrum capture
- Received NLOS signal after time and frequency synchronization and channel equalization using pilot information
  - 10 MHz (600 sub-carriers)
  - 6 frames (840 symbols): 60 ms
- Signal Level per Resource Element



- Time fading



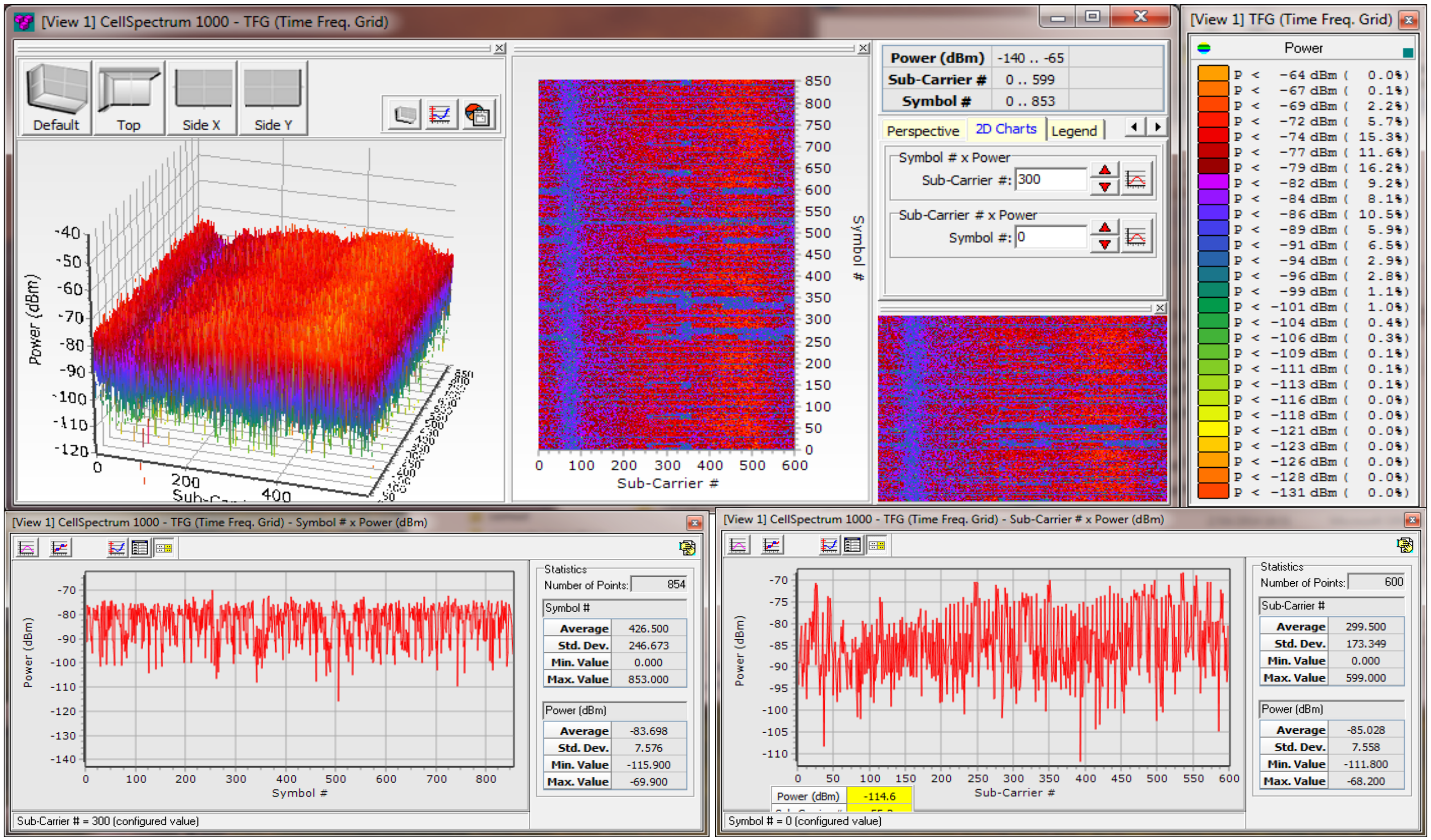


- Frequency fading

LTE Received Frame (Stationary)

- CellSpectrum capture
- Received NLOS signal after time and frequency synchronization
  - 10 MHz (600 sub-carriers)
  - 6 frames (840 symbols): 60 ms
- Signal Level per Resource Element

Time fading

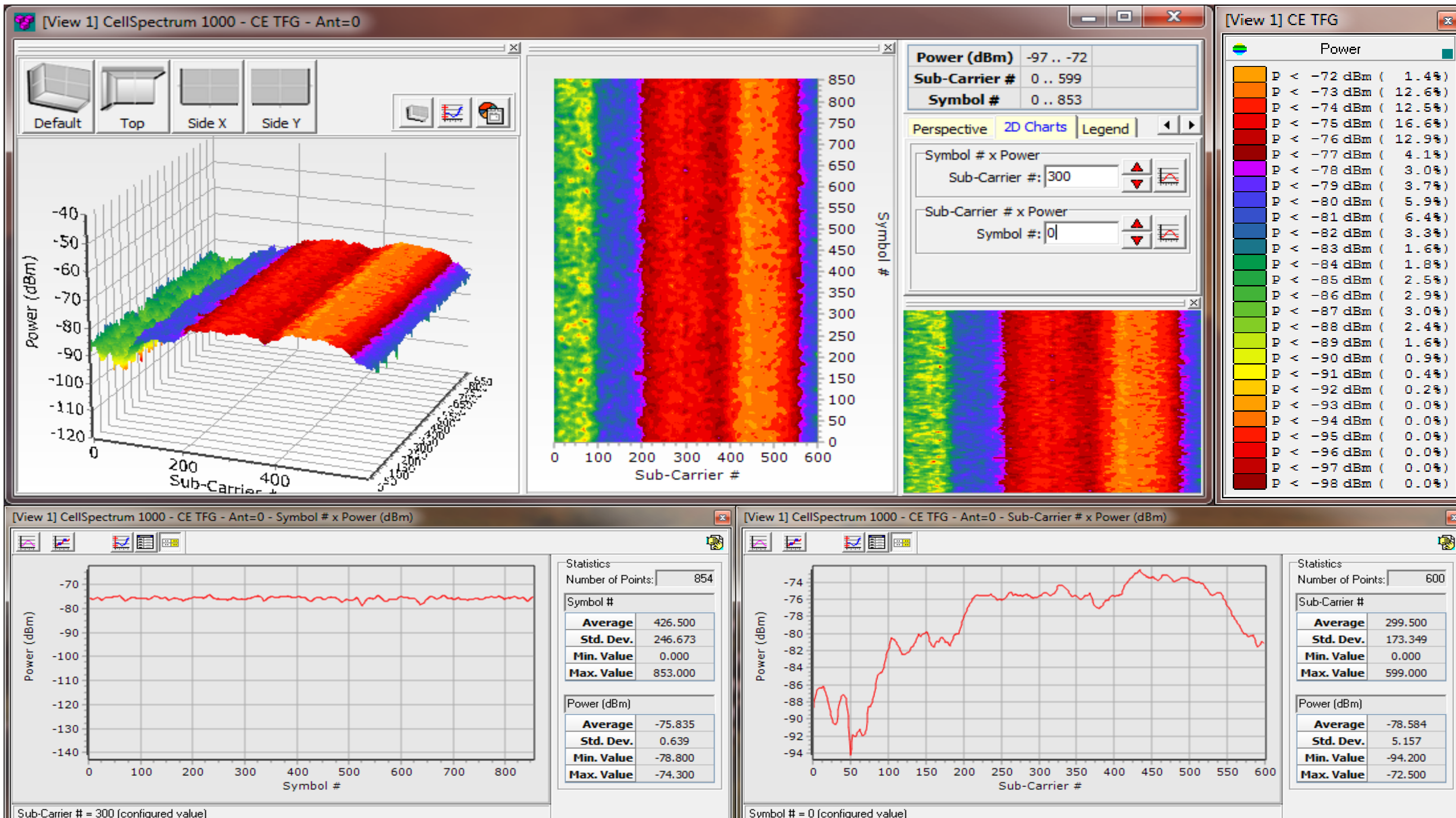


Frequency fading

LTE RF Channel NLOS (stationary)

- CellSpectrum capture
- Received NLOS signal after time and frequency synchronization
  - 10 MHz (600 sub-carriers)
  - 6 frames (840 symbols): 60 ms
- Signal Level per Resource Element

Time fading



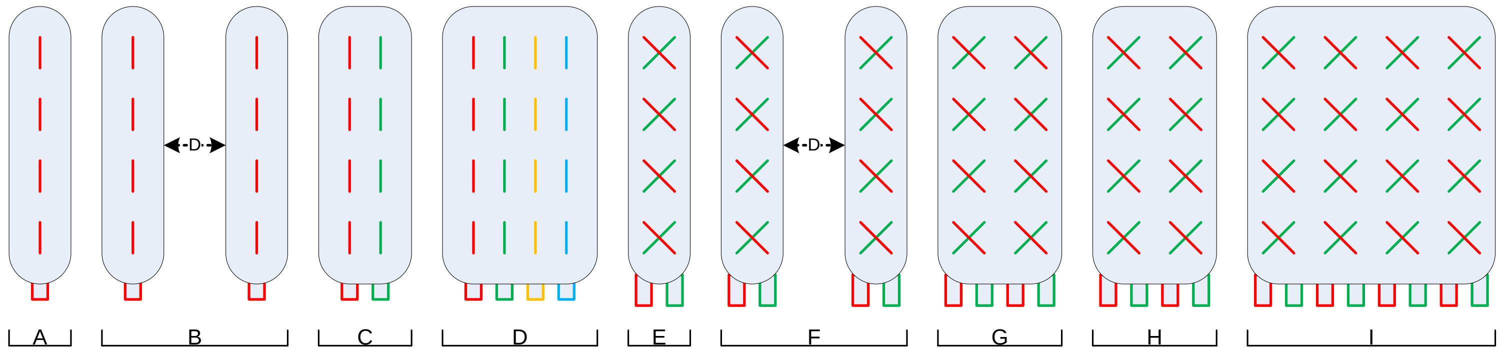
Frequency fading

3. Antenna Ports

Physical Antennas
Antenna Ports
MIMO
Transmission Modes

Physical Antennas

- Physical antennas are generally built from interconnected dipoles
- Antennas have different radiation patterns, which are characterized in the vertical and horizontal plane
- These patterns can be modified by mechanical or electrical tilts
- Antenna systems add an additional dimension to a communication channel, as they are made from multiple antennas, providing space diversity to the RF channel
- Algorithms are then used to process transmit and receive signals to maximize the connection capacity
- Using multiple antennas creates a deployment complexity, so antenna housings are built that enclose several antennas



Physical Antennas

- A signal to be transmitted can be connected to several antennas, so their radiations can be seen differently at the receiver, providing transmit diversity
  - Multiple receivers can complement the transmit diversity
- A transmitted signal arrives at a receiver pursuing multiple paths, which result s in a single signal that fluctuates around an average value
  - These variations are called fading and vary with frequency and time.
- Additional antennas create additional paths that add to the original path and create a composite signal that can enhance or impair the resulting signal
  - It is very important to model the exact location of antennas before deploying them
  - Just adding antennas does not guarantee signal improvement and may have a contrary effect
- Antennas can produce orthogonal signals by using orthogonal polarities
  - This works well in an anechoic chamber or with highly directional point to point antennas
  - When using point to multipoint antennas the orthogonality is lost due to reflections in the multipath
  - This is evident in traditional cellular in which the base station transmits with vertical polarization while a phone placed horizontally will have a horizontal polarization, but still be able to receive the transmitted signal
- Practical LTE deployments use 45° cross polarized antennas to provide diversity and some degree of orthogonality
- An antenna system may have several physical antennas connected transmitting or receiving a signal
- The same antenna system can have its antennas configured differently for different UEs or even for different moments in time for the same UE
  - Each symbol can have different antenna configurations

Antenna Ports

- The RF channel between the transmit antennas and the receive antennas has to be characterized and equalized to extract the signal
  - This is done by transmitting reference signals together with the information signal
- Antenna ports are virtual antennas characterized by the reference signal being sent
- The same set of physical antennas will be labeled with different port numbers if they are characterized by a different reference signal

Antenna Port	3GPP Release	Reference Signal	Applications
0 to 3	8	Cell specific (CRS)	Single stream transmission, transmission diversity, MIMO
4	8	MBSFN (MRS)	Multimedia Broadcast Multicast Service (MBMS)
5	8	UE specific (UERS)	Beamforming without MIMO
6	9	Position (PRS)	Location based services
7 to 8	9	UE specific (UERS)	Beamforming with MIMO; multi-user MIMO
9 to 14	10	UE specific (UERS)	Beamforming with MIMO; multi-user MIMO
15 to 22	10	CSI	Channel State Information (CSI) report

Uplink Antenna Port Number				
Channels and Signals	Antenna port index	Number of antennas		
		1	2	4
PUSCH and DMRS-PUSCH	0	10	20	40
	1		21	41
	2			42
	3			43
SRS	0	10	20	40
	1		21	41
	2			42
	3			43
PUCCH and DMRS-PUCCH	0	100	200	
	1		201	

Multiple Input Multiple Output (MIMO)

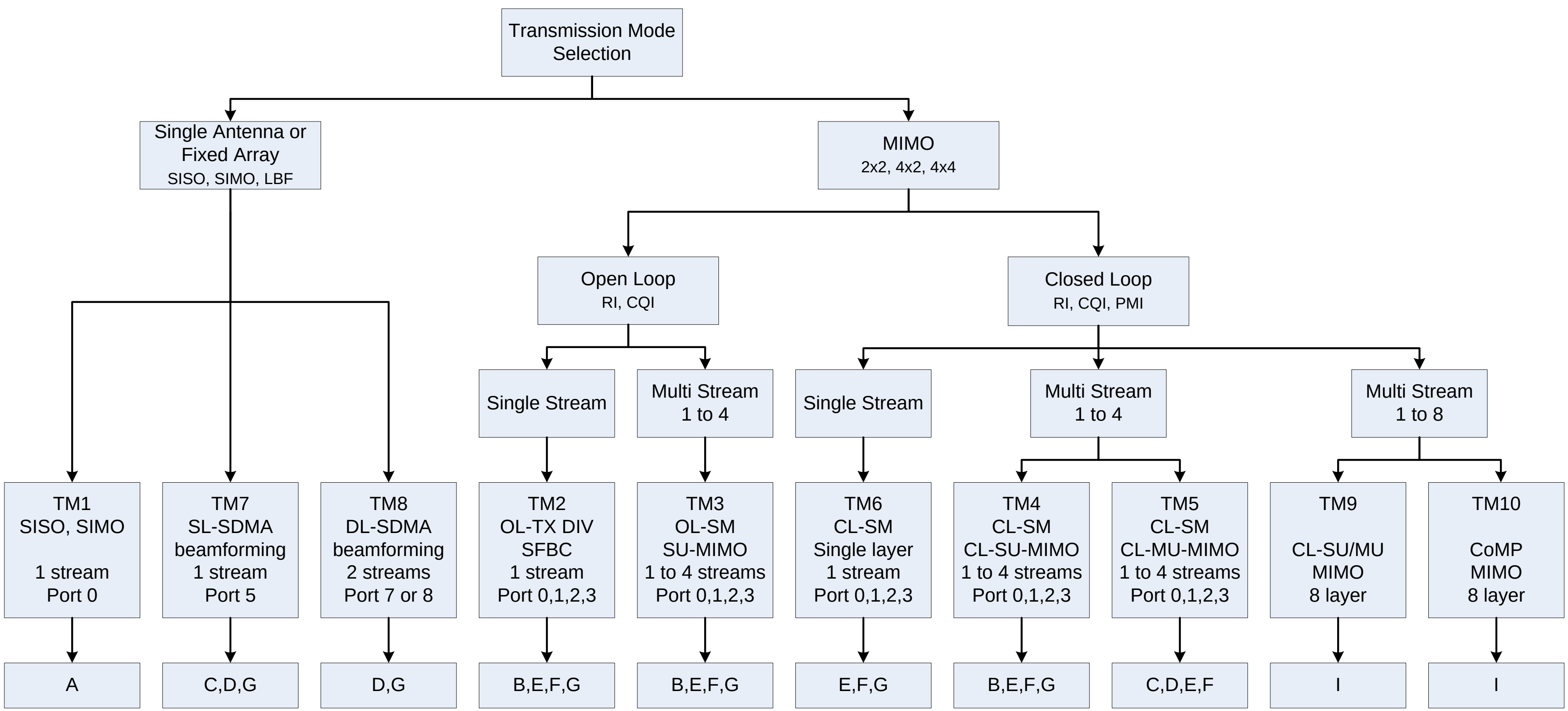
- MIMO refers to the input and output to the RF channel
- MIMO T x R, means that there are T physical transmit antennas and R receive antennas
- 3GPP divides MIMO in three categories
  - Single Antenna Transmission: Relies on receiver diversity to combat fading
  - Transmission Diversity: Provides an additional level of diversity to combat fading
  - Spatial Multiplexing: Increases capacity by sending different information in different transmission layers
    - Open Loop: UE sends Rank Indicator (RI) and Channel Quality Indicator (CQI)
    - Closed Loop: UE sends Rank Indicator (RI), Channel Quality Indicator (CQI) and Precoding Matrix Indicator (PMI)
    - Multi-user:
- In the downlink the following configurations are considered
  - 2x2 (release 8)
  - 4x4 (release 8)
  - 8x8 (release 10)
- In the uplink the following configurations are considered:
  - 1x1 (release 8)
  - 2x2 (release 10)
  - 4x4 (release 10)

4. Transmission modes

Transmission Modes

- The eNB and the cell negotiate a Transmission mode
- Each transmission mode supports two modulation schemes that are used accordingly to the type of information being sent and the RF channel conditions
- The transmission mode can be changed according to the channel conditions
- Each channel mode is better supported by certain antenna arrangements

Downlink Transmission modes



Transmission Modes DL

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- The eNB and the cell negotiate a Transmission mode
- Each transmission mode supports two modulation schemes that are used accordingly to the type of information being sent and the RF channel conditions
- The transmission mode can be changed according to the channel conditions
- Each channel mode is better supported by certain antenna arrangements

TM	Description	ANTENNAS	CONFIGURATION	RANK	PORT	SISO	SIMO	MISO	MIMO	TD	SM	BF	BF AoA	OL	CL	SU	MU
1	Single Transmit	1	A	1	0	x	x									x	
2	Transmit Diversity	2 or 4	B,E,F,G	1	0 to 3			x		x				x		x	
3	Spatial Multiplexing	2 or 4	B,E,F,G	2 or 4	0 to 3			x	x		x			x		x	
4	Spatial Multiplexing	2 or 4	B,E,F,G	2 or 4	0 to 3			x	x		x				x	x	
5	MU MIMO	2 or 4	C,D,E,F	2 or 4	0 to 3			x	x		x				x		x
6	Spatial Multiplexing	2 or 4	E,F,G	2 or 4	0 to 3			x	x							x	
7	Single Layer Beamforming	ULA	C,D,G	1	5	x	x						x			x	
8	Dual Layer Beamforming	ULA	B,E,F,G	2	7 and 8	x	x						x			x	
9	8 Layer MU MIMO	8	I		7 to 14			x	x		x						x
10	CoMP MIMO	8	I		7 to 14			x	x		x						

PDSCH Transmission Modes (Downlink)

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- The eNB and the cell negotiate a Transmission mode
- Each transmission mode supports two modulation schemes that are used accordingly to the type of information being sent and the RF channel conditions
- The transmission mode can be changed according to the channel conditions
- Each channel mode is better supported by certain antenna arrangements

Mode	PDSCH Transmission Mode (using C-RNTI to address UE)	DCI Format	Search Space	Channel State Information (UE feedback)	Antenna Ports	Release
1	Single antenna port (port 0)	1A	Common and UE specific	CQI	0	8
		1	UE specific			
2	Transmit Diversity	1A	Common and UE specific	CQI	0, (0,1), (0,1,2,3)	8
		1	UE specific			
3	Transmit Diversity	1A	Common and UE specific	CQI, RI	0, (0,1), (0,1,2,3)	8
	Open Loop Spatial Multiplexing or Transmit Diversity	2A	UE specific			
4	Transmit Diversity	1A	Common and UE specific	CQI, RI, PMI	0, (0,1), (0,1,2,3)	8
	Closed Loop Spatial Multiplexing or Transmit Diversity	2	UE specific			
5	Transmit Diversity	1A	Common and UE specific	CQI, PMI	0, (0,1), (0,1,2,3)	8
	Multi-user MIMO	1D	UE specific			
6	Transmit Diversity	1A	Common and UE specific	CQI, PMI	0, (0,1), (0,1,2,3)	8
	Closed Loop Spatial Multiplexing using Single Transmission Layer	1B	UE specific			
7	Single antenna port (port 0) or Transmit Diversity	1A	Common and UE specific	CQI	0, (0,1), (0,1,2,3) 5	8
	Single antenna port (port 5)	1	UE specific			
8	Single antenna port (port 0) or Transmit Diversity	1A	Common and UE specific	CQI (PMI, RI if instructed by eNB)	0, (0,1), (0,1,2,3)	9
	Dual layer transmission or single antenna port (port 7 and 8)	1B	UE specific		7,8	
9	Single antenna port (port 0) or Transmit Diversity	1A	Common and UE specific	CQI (PTI, PMI, RI if instructed by eNB)	0, (0,1), (0,1,2,3)	10
	Up to 8 layers transmission (port 7 to 14)	2C	UE specific		(7,8,9,10), (7 to 14)	
10	Single antenna port (port 0) or Transmit Diversity	1A	Common and UE specific	CQI PTI, PMI, RI if instructed by eNB	0	11
	Up to 8 layer transmission, ports 7-14 or single-antenna port, port 7 or 8	2D	UE specific		7-14, 7 or 8	

PUSCH Transmission Modes (Uplink)

- The eNB and the cell negotiate a Transmission mode
- Each transmission mode supports two modulation schemes that are used accordingly to the type of information being sent and the RF channel conditions
- The transmission mode can be changed according to the channel conditions
- Each channel mode is better supported by certain antenna arrangements

<table>
<tr><th>Mode</th><th>PUSCH Transmission Scheme</th><th>DCI Format</th><th>Search Space</th><th>Antenna Ports</th><th>Release</th></tr>
<tr><td>1</td><td>Single Antenna port</td><td>0</td><td>Common and UE specific</td><td>10</td><td>8</td></tr>
<tr><td rowspan="2">2</td><td>Single Antenna port</td><td>0</td><td>Common and UE specific</td><td>10</td><td rowspan="2">10</td></tr>
<tr><td>Closed Loop Spatial Multiplexing</td><td>4</td><td>UE specific</td><td>(20,21), (40,41,42,43)</td></tr>
</table>

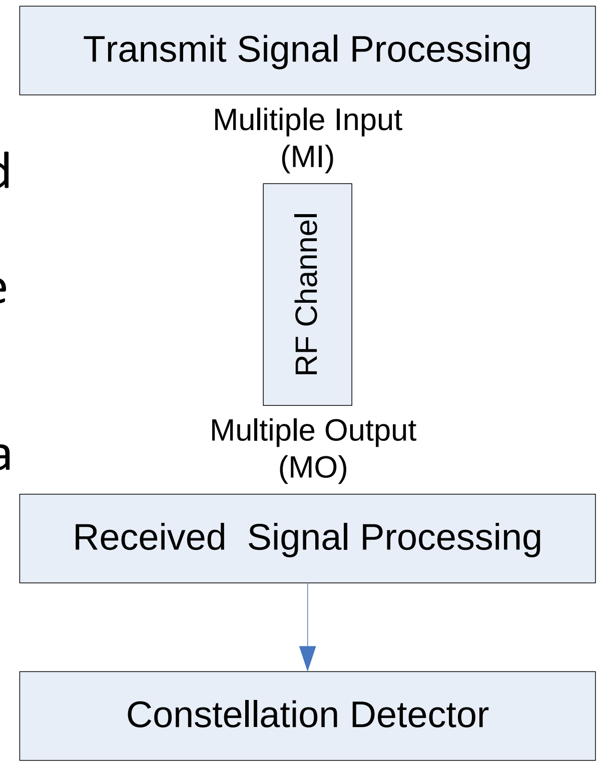
5. MIMO

Multiple Input

Multiple Output

Multiple Antenna Arrangements

- A wireless system provides communications between Base Stations (BS) and Subscriber Stations (SS) or Mobile Stations (MS)
- Base Stations allow for the installation of multiple antennas with large separations between them
- Subscriber Stations are more restricted in this sense and Mobile Stations even more so
- Multiple antenna systems are classified according to the number of antennas at the transmitter (RF channel inputs) and receiver (RF channel outputs)
- The use of multiple antennas implies in addition of extra processing on one or both sides of the RF channel
- It must be also noted that each direction (downlink and uplink) may be configured with different solutions
- The nomenclature used to classify the channels does refer to the channel (air) between transmit and receive antennas
- The input (MI) defines how many signals are sent through the air, while the output (MO) defines how many signals are received from the air



5.1 SISO

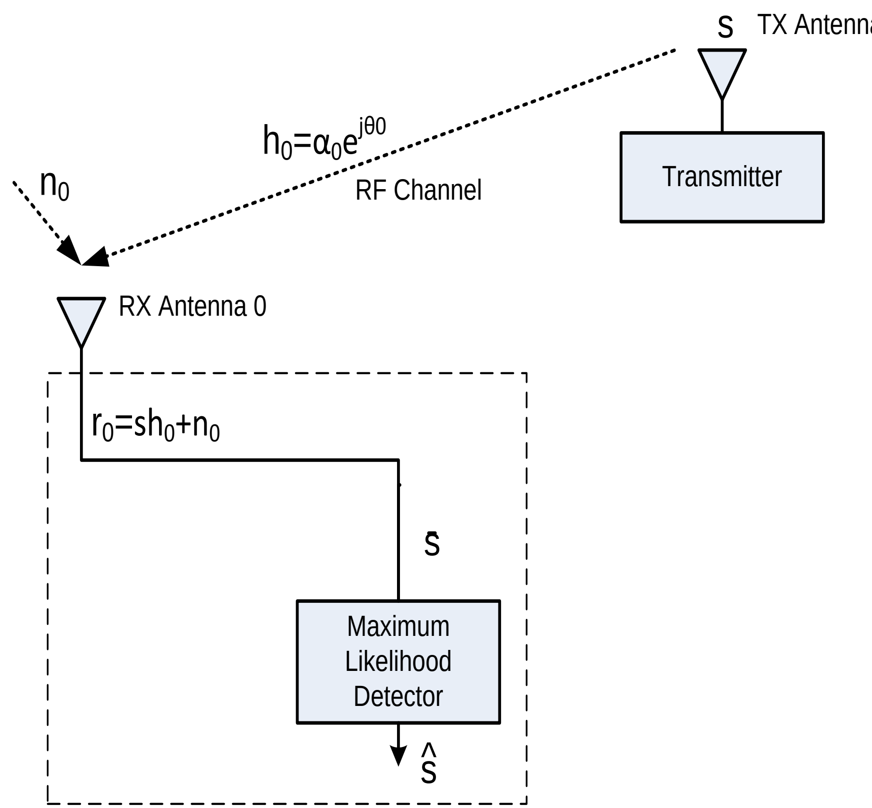
Single In Single Out

SISO- Single In to Single Out

- Traditionally, a wireless link had one transmit and one receive antenna, which can be described as a SISO (Single-In signal and Single Out signal) configuration in relation to the wireless channel
- In SISO, multipath signals are received by the antenna with the combined signal being subject to fading
- Signals have high coherence- High k factor: >10
  - Multipath signal follows a Gaussian distribution
- Signals have medium coherence- Medium k factor: 10<k>2
  - Multipath signal follows a Ricean distribution
- Signals have low coherence- Low k factor: k<2
  - Multipath signal follows a Rayleigh distribution
- The k factor can be estimated on a pixel basis from the geographical data (topography and morphology) by RF prediction tools

SISO

- The transmitted symbol "s" can be recovered if the channel response "h_0" is known
- Receive signal "r_0" is a combination of several paths (multipath)
  - The receiver aligns itself to the combined signal
  - With this I and Q will be aligned to it
- Channel response can be obtained by analyzing the Reference Signal (pilot)
  - Reference Signal pattern is known
  - Noise will distort the channel response detection
- In SISO the received signal is subject to fading caused by multipath
  - This will cause errors, of which some will be corrected by FEC
- Maximum Likelihood Detector (MLD) will select the constellation point that better represents the received signal



$r = sh_0 + n_0$	SISO

Receiver Constellation Detection

- The combined signal has to be analyzed and the constellation state detected
- This detectors perform a Maximum Likelihood Detector (MLD)
- Several sub-optimal methods were developed, including:
  - Successive Interference Cancelation (SIC) receivers make the decision on one symbol and subtract its effect to decide on the other symbol
    - This leads to error propagation
  - Sphere detectors (SD) reduce the number of symbols to be analyzed by the ML detector, by performing the analysis in stages
    - It may preserve the optimality while reducing complexity
  - Linear Detectors
    - Zero forcing (ZF) detectors invert the channel matrix and have small complexity but perform badly at low SNR
    - Minimum Mean Square Error (MSSE) detectors reduce the combine effect of interference between the channels and noise, but require knowledge of the SNR, which can only be roughly estimated at this stage
  - More Advance Detectors
    - Decision Feedback (DF) or Successive Interference Cancelation (SIC) receivers make the decision on one symbol and subtract its effect to decide on the other symbol, reducing the Inter Symbol Interference (ISI)
      - This leads to error propagation
  - Nearly Optimal Detectors
    - Sphere detectors (SD) reduce the number of symbols to be analyzed by the ML detector, by performing the analysis in stages
      - It may preserve the optimality while reducing complexity

Euclidean Distance

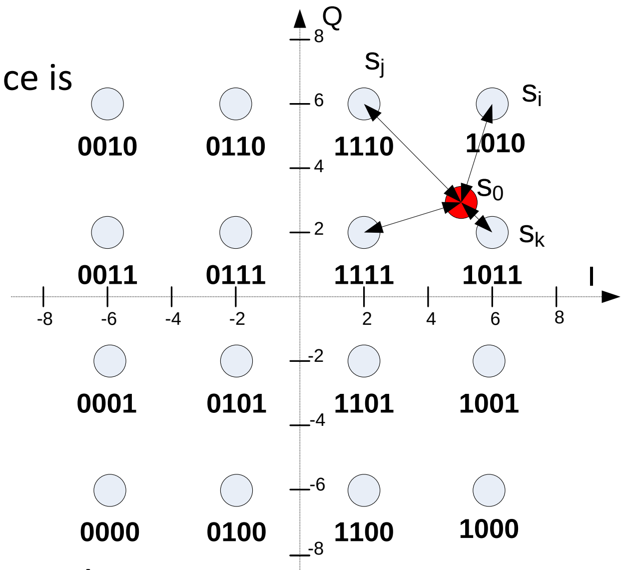
- In mathematics, the **Euclidean distance** or **Euclidean metric** is the "ordinary" distance between two points that one would measure with a ruler, and is given by the Pythagorean formula
  - In the Euclidean plane, if **p** = (p_1, p_2) and **q** = (q_1, q_2) then the distance is given by

$$d(i,q) = \sqrt{(i_1 - q_1)^2 + (i_2 - q_2)^2}$$

- Squared Euclidean distance
  - The standard Euclidean distance can be squared in order to place progressively greater weight on objects that are farther apart

$$d^2(i,q) = ((i_1 - q_1)^2 + (i_2 - q_2)^2$$

- Minimum Constellation Distance
  - The minimum distance (d) between a detected vector s_0 and constellation points s_i...s_k is given by equation below
  - The Maximal Likelihood Detector (MLD) Searches for the minimum constellation distance
  - This procedure becomes more complex as the minimum distance has to be calculated considering that different symbols have been received from more than one antenna



$d^2(s_0, s_i) \leq d^2(s_0, s_k)$	**Constellation distance**

Maximal Likelihood Detector (MLD)

- This detector verifies the distance between the received signal and the possible constellation values, by calculating the Euclidian distance between the received signal and the possible constellation states
- The Euclidean distance is the “ordinary” distance between constellation points of a modulation and the received value
- This is an NP-hard (Non-deterministic Polynomial Time –Hard) problem
- The number of possible states can be very high, when high modulations are used, and the detection is done over multiple combinations of s symbols
  – A QPSK with a single symbol results in 2^2=4 complex operations
  – A 16QAM with a single symbol results in 2^4= 16 complex operations
  – A 64QAM with a single symbol results in 2^6=64 complex operations
  – A 64QAM with two symbols results in 2^{12}=4,096 complex operations
  – A 64QAM with four symbols results in 2^{24}=16,777,816 complex operations

Multiple Antennas

- The techniques described next apply to both DL and UL directions, although it is difficult to use multiple antennas in portable phones, and if implemented the antennas are generally not fully uncorrelated
- Multiple antenna techniques rely on the existence of different paths between antennas to eliminate fading. Signals with similar fading characteristics are said to be coherent, whereas signals with different fading are not coherent (or diverse).
- Co-located antennas have to be optimized to provide the desired signal diversity (no coherence), which can be done by adjusting the position or azimuth angle of the antennas. Spacing of at least $\lambda/2$ (half a wavelength) and an angular shift of at least 1/8 of the antenna beamwidth may optimize signal diversity.
- Optimizing the antennas position is not sufficient to assure non coherent signals, as the amount of LOS components in relation to indirect components has a large influence on signal coherence. LOS paths tend to be coherent, whereas non LOS paths tend to be non coherent. The amount of LOS present in a multipath signal is defined in a Ricean distribution by the k factor, which is defined as the ratio of the dominant component's signal power over the (local-mean) scattered components power.

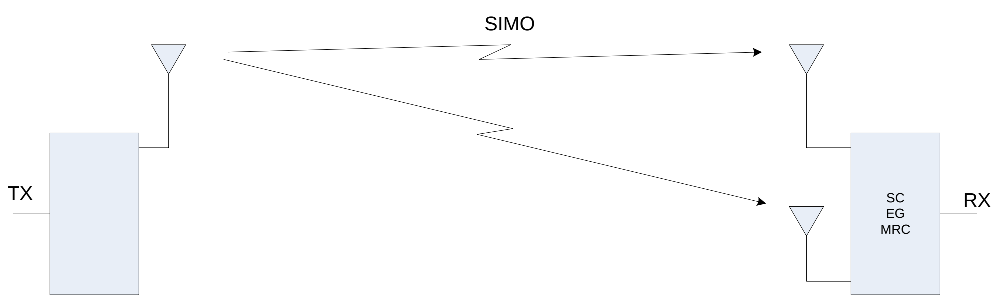
5.2 SIMO

Single In

Multiple Out

SIMO- Single In to Multiple Out

- Adding additional receive antennas creates alternative paths that will receive different multipath components and consequently be subject to different fading instances
  - The difference in the resulting signals is defined by the correlation factor
- This setup is called a SIMO (Single-In signal and Multiple-Out signals) configuration, and is also known as Receive Diversity
  - The multiple output signals from the RF channel have to be combined, so a single signal is sent to the receiver
- In Receive Diversity the receiver learns information about the channel by analyzing known transmissions, such as preamble and pilots
- There are three basic techniques with which the signals can be combined before the receiver:
  - Diversity Selection Combining (DSC)- The strongest signal is always selected
  - Equal Gain Combining (EGC)- The signal are simply added together (phase correction may be applied)
  - Maximal Ratio Combining (MRC)- The signals are added together weighted by their SNR
- Receive diversity improves the overall SNR with the number of antennas, but for this to happen the paths should be non-coherent and the combining device optimal
  - During the network design, the path correlation should be considered as an efficiency factor to be applied to the combining device gain



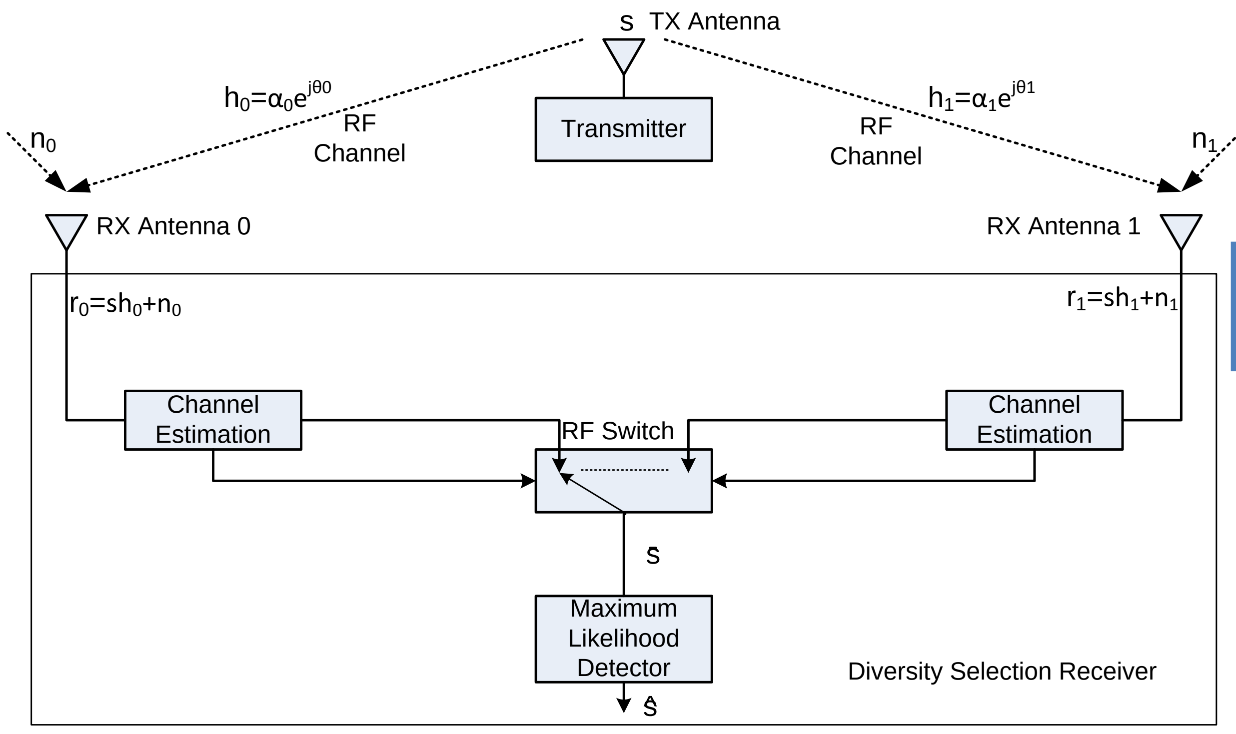
Receive Diversity

- A radio link is subject to quality variations that degrade its throughput capacity
- Adding an additional receiver provides a diverse path that may complement the information received by the first receiver and, consequently, help in recovering throughput
- Receive diversity techniques are of the SIMO type
- To better understand receive diversity, assume that a transmitter sends a signal **s**, which travels through two different RF channels
- The received signals have noise added to them in equations
- This combination of signal and noise will then be received at the other end of the communication channel
- There are three main methods commonly used to benefit from this received diversity
  - Diversity Selection Combining (DSC)
  - Equal Gain Combining (EGC)
  - Maximal Ratio Combining (MRC)

$h_0 = \alpha_0 e^{j\theta_0}$	**RF channel 1**
$h_1 = \alpha_1 e^{j\theta_1}$	**RF channel 2**
$r_0 = sh_0 + n_0$	**RF channel 1 with noise**
$r_1 = sh_1 + n_1$	**RF channel 2 with noise**

Diversity Selection (DS)

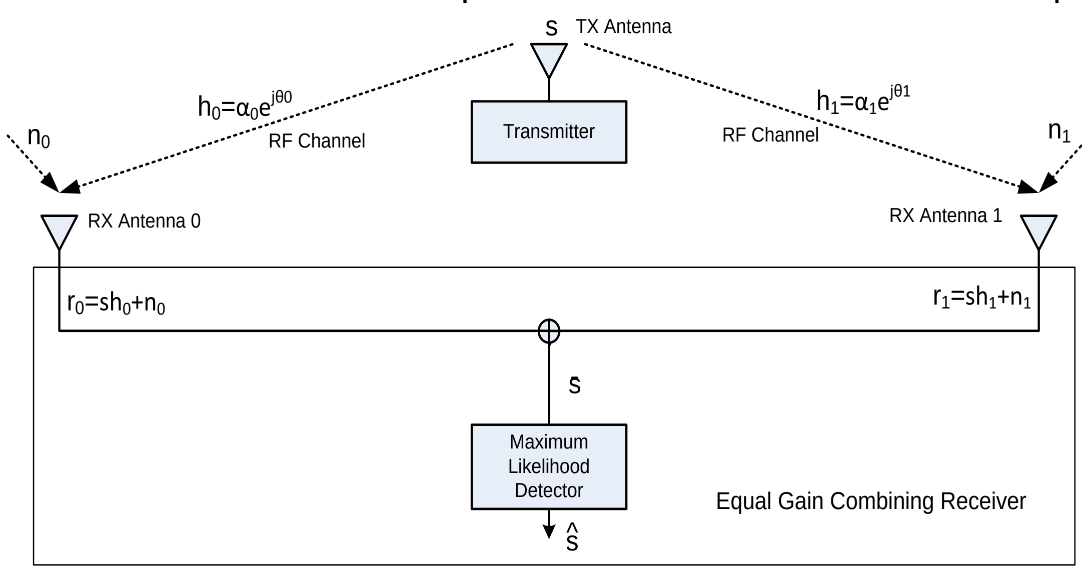
- Diversity Selection is another variation of SIMO
- In this technique, each branch analyzes known transmitted information (such as pilots) and informs its Signal to Noise Ratio (SNR) to the diversity switch, which chooses the branch with the best ratio
- This is a technique commonly adopted by WLAN systems
- In this technique, each branch analyzes known transmitted information (such as pilots) and informs its Signal to Noise Ratio (SNR) to the diversity switch, which chooses the branch with the best ratio
- This method should only be used when the channel coherence time is much larger than the symbol duration, so the best branch chosen from pilot analysis will hold for several symbols (at least until the next channel assessment); otherwise, by the time the switch is made from one antenna to the other, the channel might have changed already, thus affecting the SNR at each antenna
- There is no gain if the channels are coherent and for non coherent channels the gain can be significant if the fading does not coincide in both channels



$r = sh_0 + n_0 \quad or$ $r = sh_1 + n_1$	**Selection Combining**

Equal Gain Combining (EGC)

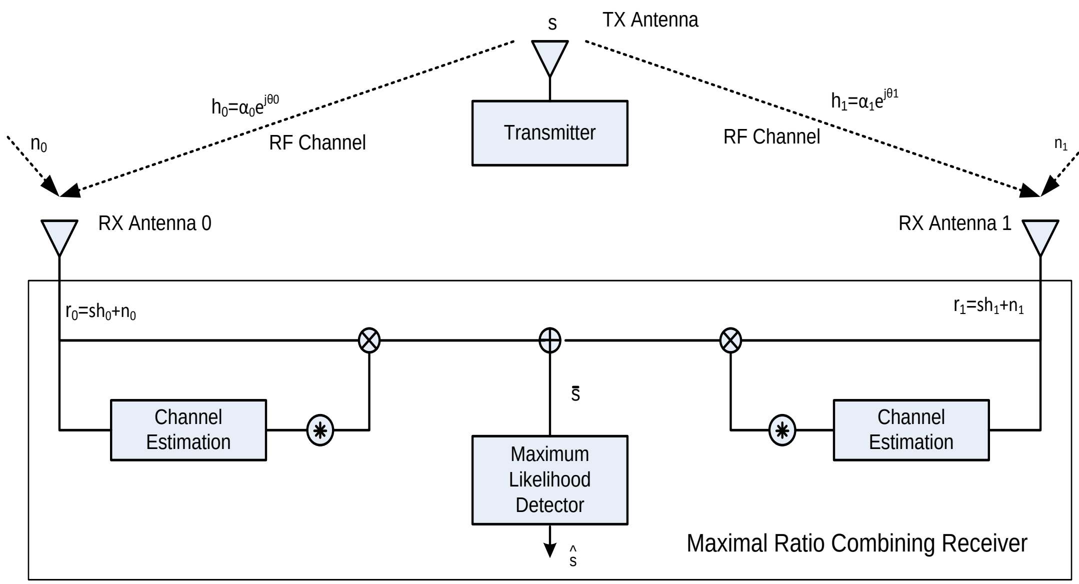
- Equal Gain Combining is illustrated below
- The signals received from both branches are combined, as expressed in the equation
  - Each branch should have its own LNA, to avoid combiner loss
- For coherent channels (identical or nearly identical), there is no real gain as the signal and the noise rise together by 3 dB, that is, even though there was an increase in the signal (added twice), there was exactly the same increase in noise, thus ir results in the same as receiving in only one antenna, i.e. no diversity
- For non coherent channels there is a significant gain when fading occurs in one channel and not in the other
- This solution is the easiest type of receive diversity to implement, but the benefit only happens when the probability of fading overlap between the channels is small
  - The use of different polarizations for the Rx antennas helps to maximize this benefit



$r = s(h_0 + h_1) + n_o + n_1$	**Equal Gain Combining**

Maximal Ratio Combining (MRC)

- Another SIMO technique is Maximal Ratio Combining
- In this method, the phase and gain of each branch is optimally adjusted prior to combining the signals
  - This requires a good knowledge of each channel, which is derived from pilot analysis
  - This method estimates each channel independently and then multiplies the signal by the convoluted channel
  - This allows the recovery of the best possible signal, although the noise can be a major impairment, as it is also amplified with the faded signal
  - The method performs well when the signals are quite above noise level
  - It is a more complex and expensive method to implement, than the two previous receive diversity options
- In this technique, in case of fading in one of the branches, a maximum gain is applied to it, which also increases the noise level
- As an example, assume that the signals are being received at -60 dBm and that the noise floor is at -90 dBm.
  - A fading of 30 dB implies on a 30 dB gain for this branch and, consequently, the noise reaches -60 dBm drastically reducing the SNR
  - A possible solution in this case would be to drop the signal that requires a higher gain, that is, a mix of MRC and DSC



$\tilde{s} = h_0^* r_0 + h_1^* r_1$	**Received signal**
$\tilde{s} = h_0^*(sh_0 + n_0) + h_1^*(sh_1 + n_1)$	**Received signal with noise**
$\tilde{s} = (\alpha_0^2 + \alpha_1^2)s + h_0^* n_0 + h_1^* n_1$	**Maximal Ratio Combining**

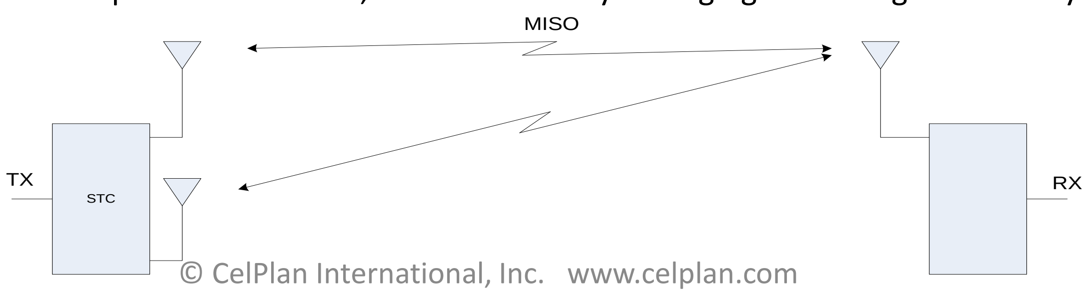
5.3 MISO

Multiple In

Single Out

MISO- Multiple In to Single Out

- Adding additional transmit antennas **creates** alternative paths that will have different multipath components and, consequently, be subject to different fading instances
  - The amount of difference in the resulting signals is defined by the coherence factor
- Multiple transmit antenna configurations are called MISO (Multiple-In signal and Single-Out signal), and are also known as Transmit Diversity
- The receive antenna receives signals coming from two or more transmit antennas, so, in principle, it receives n times the power, n being the number of transmit antennas
- This gain in power is known as the Array Gain and can be positive or negative (representing a loss) depending on the signal coherence
  - The signal coherence is a direct function of the antenna correlation
- There are two modes of transmitting schemes for MISO: open loop, and closed loop.
  - In open loop, the transmitter does not have any information about the channel
  - In closed loop, the receiver sends Channel State Information (CSI) to the transmitter on a regular basis
    - The transmitter uses this information to adjust its transmissions using one of the methods below.
- There are two main configurations for Transmit Diversity:
  - Transmit Channel Diversity- The transmitter evaluates periodically the antenna that gives best results and transmits on it
    - Only one antenna transmits a time.
  - Transmit Redundancy- A linear pre-coding is applied at the transmitter and a post-coder is used at the receiver, both using the CSI information
    - Both antennas transmit simultaneously
- Additional transmit diversity can only be obtained by replacing sets of multiple symbols by orthogonal signals and transmitting each orthogonal symbol on a different antenna
  - Alamouti proposed an orthogonal code for two symbol blocks in a method called Space Time Block Coding (STBC)
  - 3GPP specifies a similar solution but replacing time by frequency, in a method called Space Frequency Block Coding (SFBC)
  - This method does not improve overall SNR, but reduces it by averaging the fading over two symbols or two sub-carriers



Transmit Diversity

- The addition of more transmitters creates new multipath and may increase the detection options in a method known
- Transmit diversity can be represented by a matrix that relates data send in the antennas to a certain sequence of symbols
  - n :transmit antennas
  - T :Symbols(represents the transmit diversity block size)
  - s_{ij} :modulated symbol
- This matrix is defined by a code rate that expresses the number of symbols that can be transmitted on the course of one block
- A block that encodes k symbols has its code rate defined by the equation below
- The three, most common types of transmit diversity techniques are:
  - Receiver based Transmit Selection
  - Transmit Redundancy
  - Space Time Transmit Diversity
  - Space Time Block Code (Alamouti's code or Matrix A)
  - Transmit and Receive Diversity
  - Spatial Multiplexing (Matrix B)

a_{ij}

$i = row\ (Symbol)$

$j = column$ (Antenna)

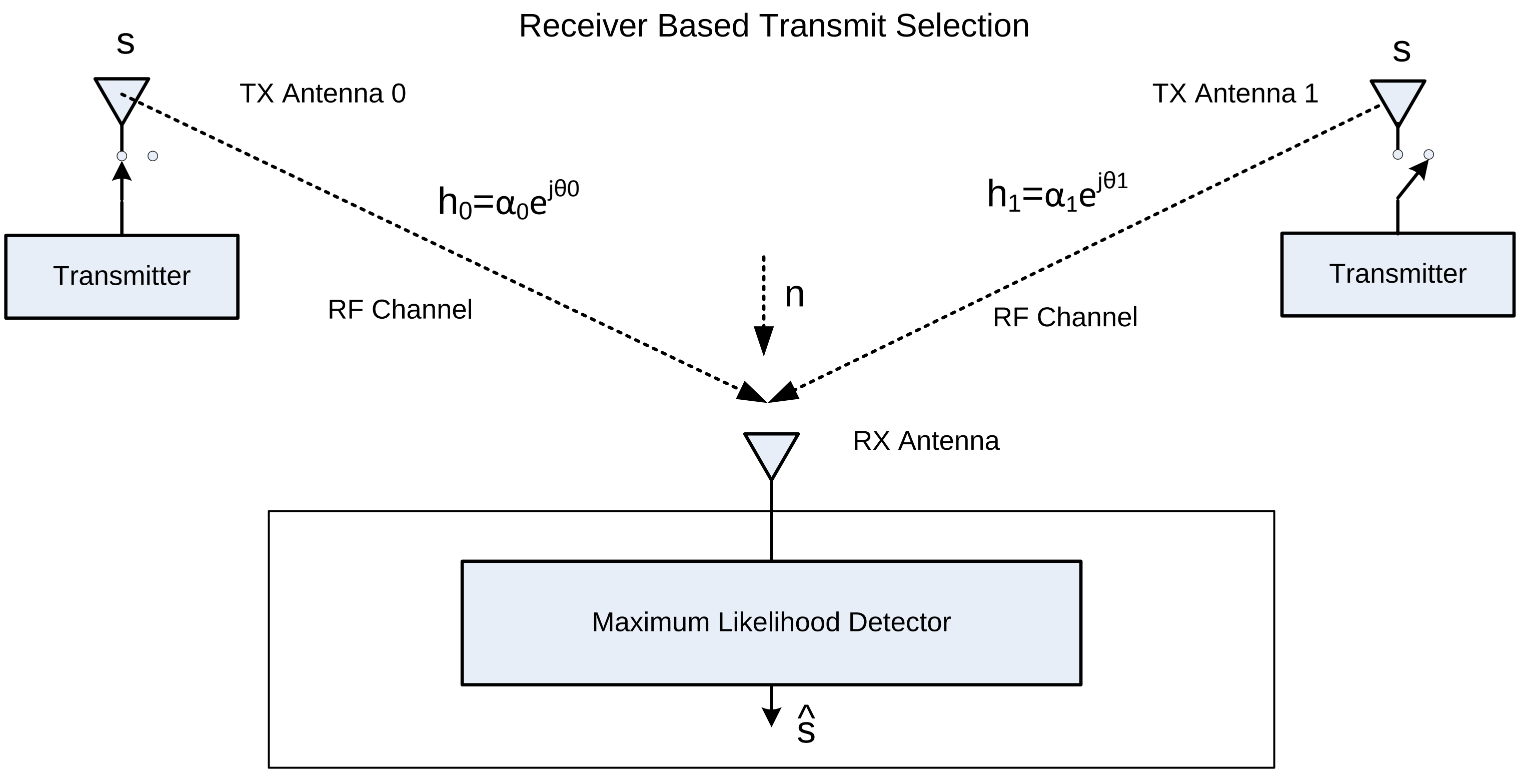
$$\begin{array}{cc} & \text{Transmit antennas} \rightarrow \\ \text{Symbols} \downarrow & \begin{bmatrix} s_{11} & \cdots & s_{1n} \\ \vdots & \ddots & \vdots \\ s_{T1} & \cdots & s_{Tn} \end{bmatrix} \end{array}$$

$r = \frac{k}{T}$	**Matrix code rate**

Receiver based Transmit Selection

- In TDD systems, transmit and receive directions use the same channel
- When channels vary very slowly, it is reasonable to assume that the best channel used for the receive side would be the best one to transmit as well
- However, this only holds true for channels with coherence time larger than the frame time

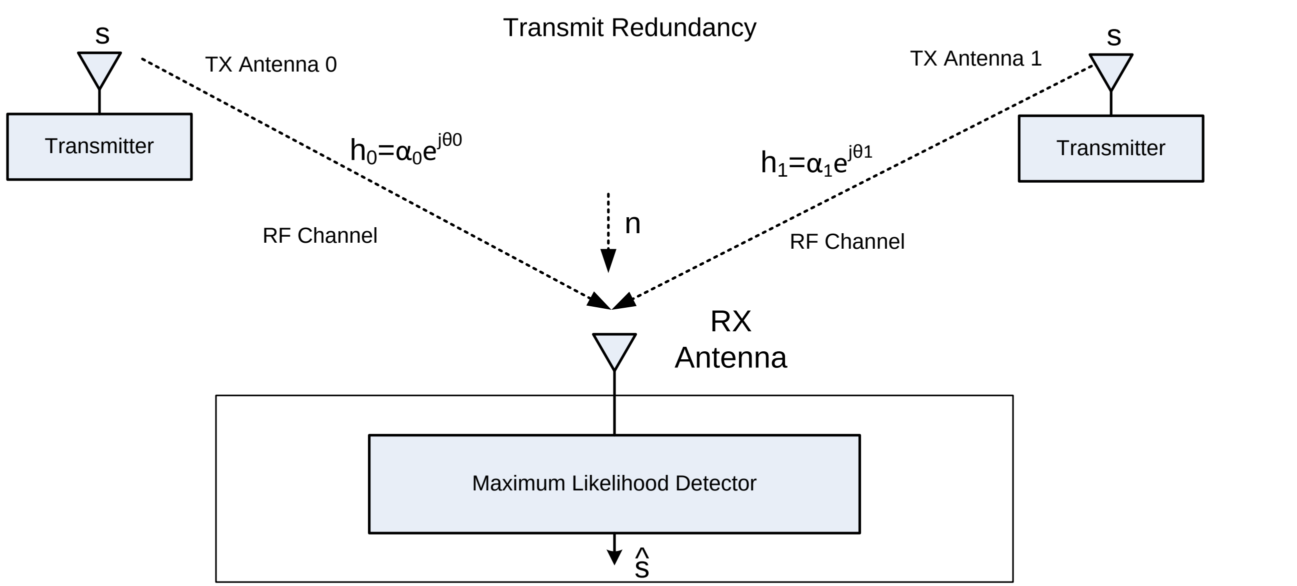
$r = s_0 h_0 + n \quad or \quad r = s_1 h_1 + n$	**Received based transmit selection**



Transmit Redundancy

- Transmission is made on both channels all the time, which requires both signals to arrive at the antennas at the same time
  - The circuits should be designed to avoid different delays in the path to the antennas and cable lengths should be exactly the same
- In coherent channels (similar channels) the received signal will increases by 3 dB, whereas in non coherent channels, the extra path reduces multipath fading but also becomes a source of interference
  - Designer should configure antennas to obtain uncorrelated channels but with a low dispersion between them
  - An example would be to use directional antennas with azimuth angle diversity (pointing antennas to slightly different angles) or use different antenna polarities
  - It is important for designers to analyze the sources of multipath before deciding on the best deployment strategy

$r = sh_0 + sh_1 + n$	**Received signal transmit redundancy**



Space Time Transmit Diversity

- Additional schemes of SIMO that add a time component to the space diversity provided by multiple antennas have also been proposed
- Some examples are:
  - Delay Diversity and
  - Space Time Trellis
- Both methods rely on creating additional multipath and are complex to implement
- A simpler method was proposed by Siavash Alamouti
- On his proposition, the multipath is delayed by a full symbol, and then a conjugate value is sent to cancel the reactive part of the signal
  - This technique is easy to implement, but requires the channel to remain stable over a period of two symbols
  - This means that the coherence time should be larger than two symbols
  - This method is called Space-Time Block Coding (STBC or STC), also known as Matrix A

Space Time Block Code-Alamouti's code (Matrix A)

- In this technique, each transmission block is made of two symbols in time
- Each antenna sends the information as depicted in the table below
  - The operations applied over the information were carefully chosen to cancel the unwanted information at each antenna
  - In this technique, even though different information is sent by each antenna on one symbol, the same information is repeated over the next symbol, thus this is still considered a diversity scheme
- This is the only type of code that can reach a coding rate of 1
  - In the WiMAX standard, this matrix is referred to as Matrix A
  - Equation below shows how the matrix is built
  - BS support of this method is mandatory in the WiMAX and LTE standards

$X = \begin{bmatrix} s_0 & s_1 \\ -s_1^* & s_0^* \end{bmatrix}$	Matrix A

Alamouti	Antenna 0	Antenna 1
Symbol 0	S_0	S_1
Symbol 1	$-S_1^*$	S_0^*

Space Time Block Code-Alamouti's code (Matrix A)

- The received signal for the first symbol (0) and the second symbol (1) are shown in equations below

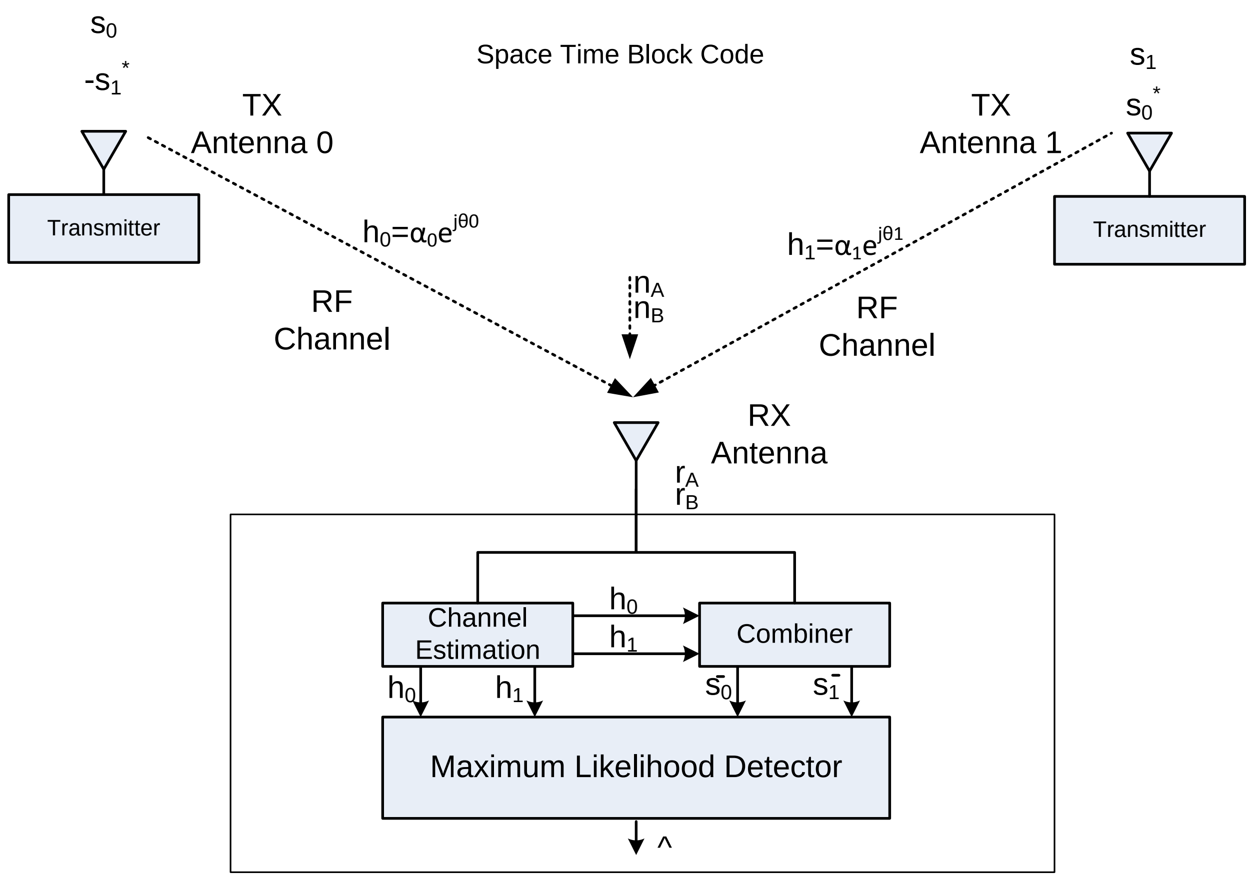
$r_0 = h_0 s_0 + h_1 s_1 + n_0$	**First symbol received signal**
$r_1 = -h_0 s_1^* + h_1 s_0^* + n_1$	**Second symbol received signal**

- We have now two RF channels present and to be able to detect them, alternate pilots should be sent by each antenna, so the receiver can estimate the channels independently
- This scheme only works if the channels are approximately constant over the period of two symbols (coherence time should be larger than two symbols)
  - Once the channels are estimated the original signals can be obtained by a simple combination of the received signals and the estimated channel responses
  - The output signals are presented below

$\check{s}_0 = h_0^* r_0 + h_1 r_1^* = ({\alpha_0}^2 + {\alpha_1}^2) s_0 + h_0^* n_0 + h_1 n_1^*$	**Space Time Block code –s_0**
$\check{s}_1 = h_1^* r_0 - h_0 r_1^* = ({\alpha_0}^2 + {\alpha_1}^2) s_1 - h_0 n_1^* + h_1^* n_0$	**Space Time Block code–s_1**

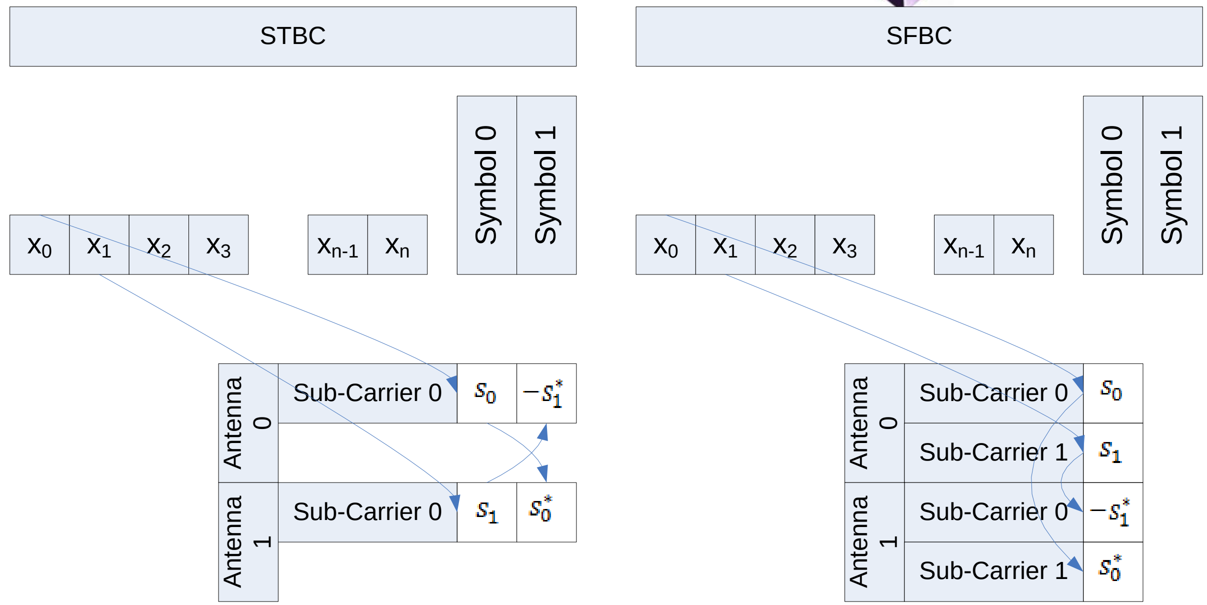
Space Time Block Code-Alamouti's code (Matrix A)

- This scheme has the same drawback of the MRC receiver, as the noise is amplified when one of the signals fades
  - The same solution suggested for MRC can be applied here

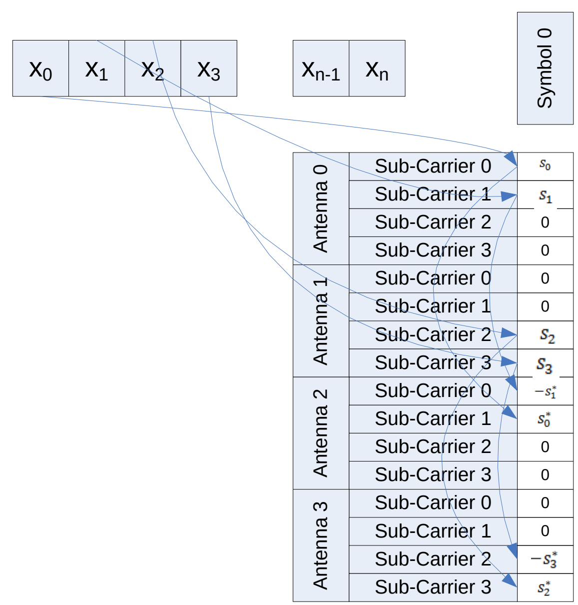


Space Frequency Block Code (SFBC)

- SFBC is similar to STBC but it replaces time by frequency

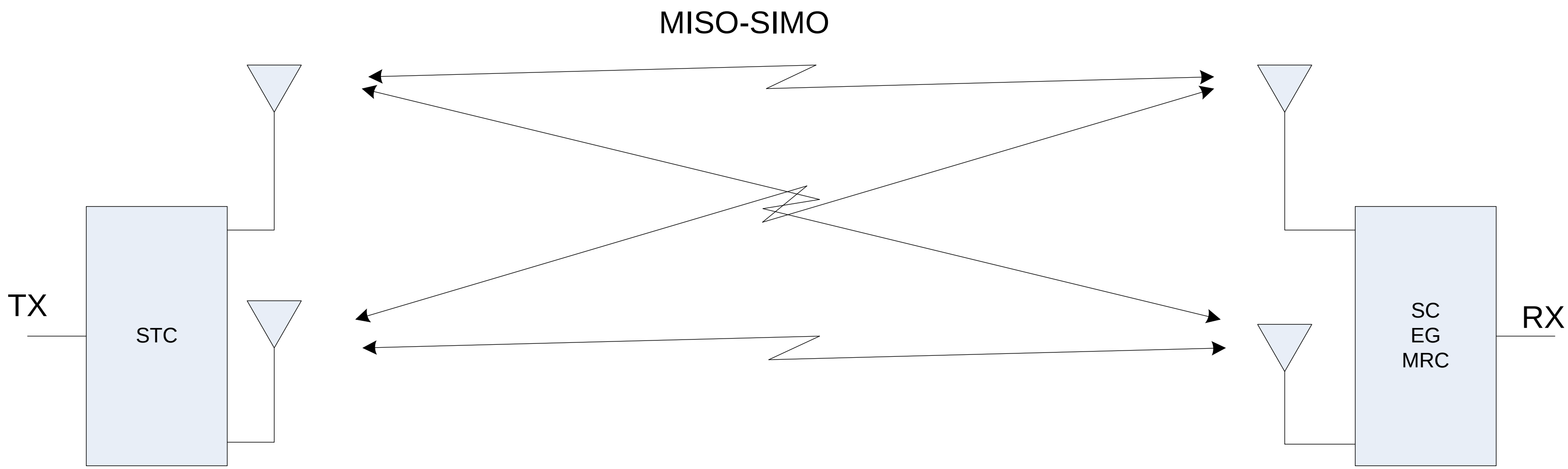


- SFBC for 4 Antenna is a simple extension of 2 antennae case



MISO-SIMO

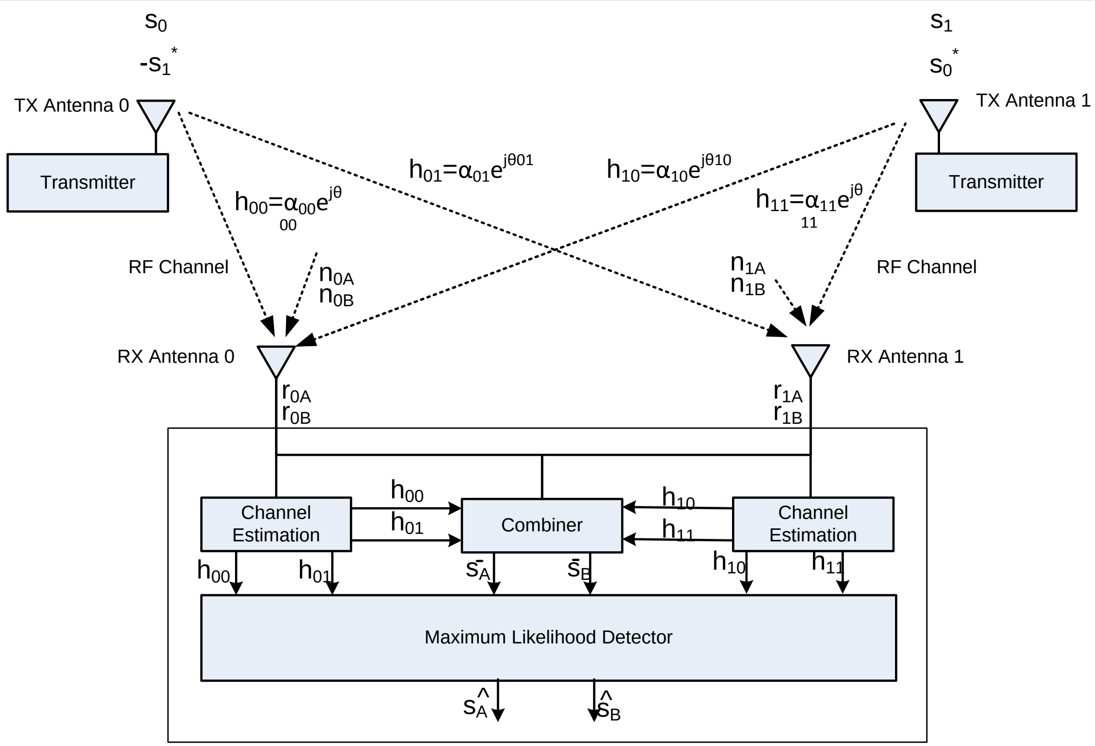
- The Transmit and Receive diversity methods described above can be combined in a Transmit and Receive diversity



Transmit and Receive Diversity (TRD)

- Transmit diversity can be mixed with MRC to provide a fourth order diversity MIMO scheme (2x2) as can be seen in figure and equations below

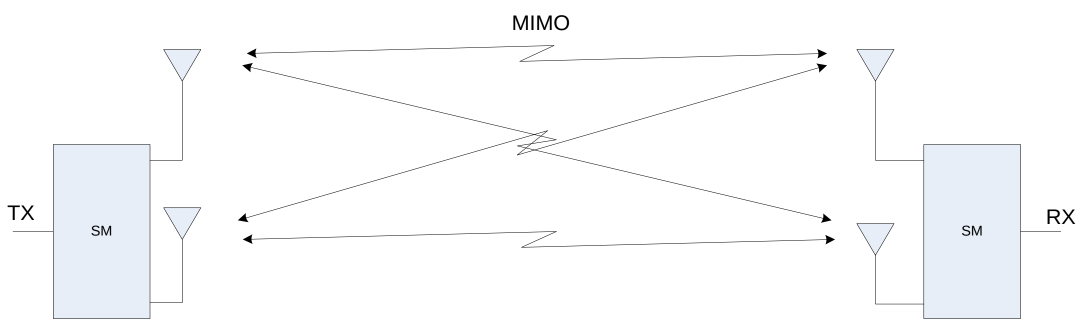
$r_0 = h_0 s_0 + h_1 s_1 + n_0$	**TRD received signal 0**
$r_1 = -h_0 s_1^* - h_1 s_0^* + n_1$	**TRD received signal 1**
$r_2 = h_2 s_0 + h_3 s_1 + n_2$	**TRD received signal 2**
$r_3 = -h_2 s_1^* + h_3 s_0^* + n_3$	**TRD received signal 3**
$\check{s}_0 = h_0^* r_0 + h_1 r_1^* + h_2^* r_2 + h_3 r_3^* = ({\alpha_0}^2 + {\alpha_1}^2 + {\alpha_2}^2 + {\alpha_3}^2) s_0 + h_0^* n_0 + h_1 n_1^* + h_2^* n_2 + h_3 n_3^*$	**TRD output signal 0**
$\check{s}_1 = h_1^* r_0 - h_0 r_1^* + h_3^* r_2 - h_2 r_3^* = ({\alpha_0}^2 + {\alpha_1}^2 + {\alpha_2}^2 + {\alpha_3}^2) s_1 - h_0 n_1^* + h_1^* n_0 - h_2 n_3^* + h_3^* n_2$	**TRD output signal 1**



5.4 MIMO

MIMO- Multiple In to Multiple Out

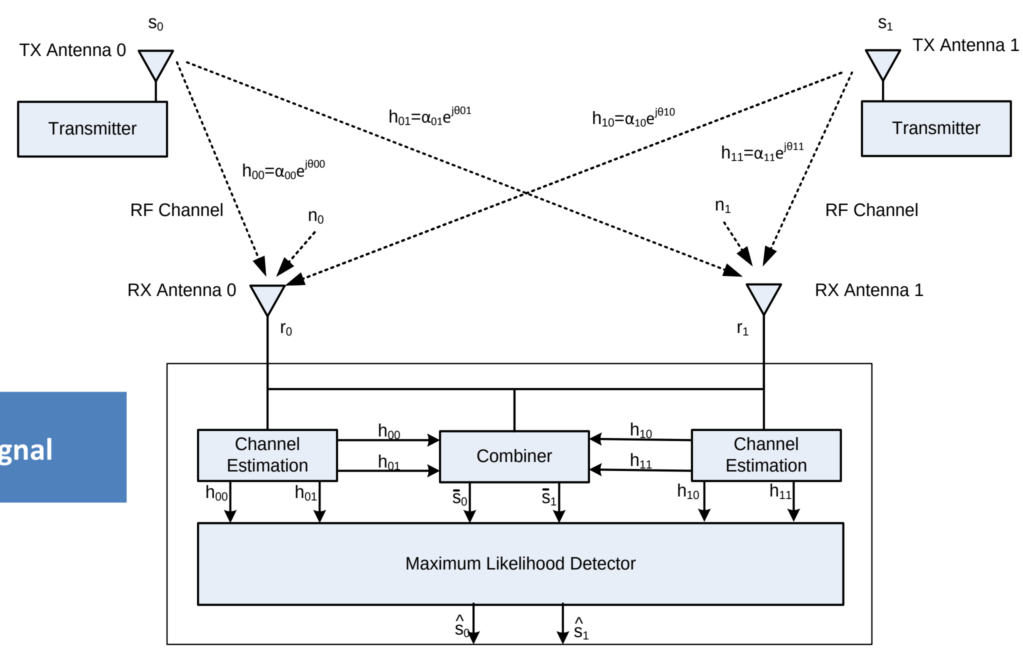
- Spatial multiplexing (SM) or MIMO (Multiple In signal and Multiple Out signal) are improvements over the previous solution that uses simultaneous transmit and receive diversity
- The use of transmit and receive diversity increases the robustness of the channel, the use of spatial multiplexing trades this robustness by capacity when the channel is robust enough
- In spatial multiplexing, each antenna transmits different data, so each receiver receives different streams of data
- A matrix is then assembled to relate each transmit signal to each receive antenna; as long as this matrix has a number of unique values equal or larger than the number of transmitted streams, it is possible to mathematically decode the data
- In principle, the throughput can be multiplied by the number of transmit antennas, but this thought can lead to confusion when trying to understand the gain given by MIMO techniques
  - This multiplication of throughput does not actually happen in real life because the channels are not completely orthogonal and interfere with each other.
- The SNIR requirement at each receive antenna is higher than if only one transmission was being performed, which implies in choosing a modulation scheme with a lower throughput
  - So, although the nominal throughput of the link is multiplied by the number of antennas, it is reduced due to the higher SNIR requirement (which forces use of a lower modulation scheme)
  - The final result is between both values and depends largely of the antenna correlation.
- In open loop, the transmitter is unaware of the channel
- In closed loop, the receiver can recover some channel information and applies one of the methods below
  - Linear Detectors (LD) - the decoder applies the inverse of the channel to amplify it, trying to remove the channel distortions
  - Interference Cancellation (BLAST- Bell Labs Layered Space Time) - this technique adds another level of randomness by circulating the data through the different antennas, providing space diversity additionally to time diversity
  - SVD (Single Value Decomposition) pre-coding and post-coding - the diagonalization done in BLAST can be done by applying the channel knowledge
  - Linear Pre-Coding and Post-Coding (LPCP) - channel knowledge allows decomposing the channel model in a set of parallel channels and more power is directed to the channels with more gain. The reverse operation is done at the receiver.



Spatial Multiplexing (SMX)

- It is also possible to increase network capacity by sending different information from each transmit antenna
- This is the case of Spatial Multiplexing (also known as Matrix B or Matrix C, or commonly referred to, albeit incorrectly, as MIMO B or MIMO C)
  - MIMO B transmits different data from 2 antennas
  - MIMO C transmits different data from 4 antennas
- Matrix B and C nomenclature is used in WiMAX, LTE uses 2x2 and 4x4
- In this technique there is no diversity, as the information transmitted by each antenna is different
- Each channel response is estimated using alternate pilots for each transmitter
- It is possible to increase network capacity by sending different information from each transmit antenna

$X = [s_1 \quad s_2]$	**Matrix B**
$r_0 = h_{00}s_0 + h_{10}s_1 + n_0$	**Matrix B receive 0**
$r_1 = h_{01}s_0 + h_{11}s_1 + n_1$	**Matrix B receive 1**
$\begin{pmatrix} r_0 \\ r_1 \end{pmatrix} = \begin{pmatrix} h_{00} & h_{10} \\ h_{01} & h_{11} \end{pmatrix}\begin{pmatrix} s_0 \\ s_1 \end{pmatrix} + \begin{pmatrix} n_0 \\ n_1 \end{pmatrix}$	**Matrix B received signal**



Conditioning of Linear Equations

- A set of linear equations can be well or ill conditioned
  - A well conditioned set of equations will give similar results when its parameters suffer small variations (are affected by noise)
  - An ill conditioned set of equations will give disproportional results when its parameters suffer small variations (are affected by noise)
- Ill conditioned linear equation

$$[H] \quad [s] = [r]$$

$$\begin{bmatrix} 1 & 2 \\ 2 & 3.999 \end{bmatrix} \begin{bmatrix} 4 \\ 8 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ 2 & 3.999 \end{bmatrix} \begin{bmatrix} 4.001 \\ 7.998 \end{bmatrix} = \begin{bmatrix} -3.999 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} 1.001 & 2.001 \\ 2.001 & 4 \end{bmatrix} \begin{bmatrix} 4 \\ 7.999 \end{bmatrix} = \begin{bmatrix} 6.98902 \\ -1.49725 \end{bmatrix}$$

- Well conditioned linear equation

$$[H] \quad [s] = [r]$$

$$\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 7 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 4.001 \\ 7.001 \end{bmatrix} = \begin{bmatrix} 1.999 \\ 1.001 \end{bmatrix}$$

$$\begin{bmatrix} 1.001 & 2.001 \\ 2.001 & 3.001 \end{bmatrix} \begin{bmatrix} 4 \\ 7 \end{bmatrix} = \begin{bmatrix} 2.003 \\ 0.997 \end{bmatrix}$$

Condition Number

- In the field of numerical analysis, the **condition number** of a function with respect to an argument measures how much the output value of the function can change for a small change in the input argument
  - This is used to measure how sensitive a function is to changes or errors in the input, and how much error in the output results from an error in the input
  - A problem with a low condition number is said to be **well-conditioned**, while a problem with a high condition number is said to be **ill-conditioned**. The condition number is a property of the problem
- A system of equations is considered to be **well-conditioned** if a small change in the coefficient matrix or a small change in the right hand side results in a small change in the solution vector
- A system of equations is considered to be **ill-conditioned** if a small change in the coefficient matrix or a small change in the right hand side results in a large change in the solution vector
- Every invertible square matrix has a condition number and coupled with the machine epsilon, it is possible to quantify how many digits can be trusted in the solution

Condition Number Calculation

- A single Condition Number (CN) can be used to define if a set of equations is well or ill conditioned
  - A CN lower than 10 dB indicates well conditioned set of equations
  - A CN larger than 10 dB indicates well conditioned set of equations
- Norm of a matrix is the maximum value of the sum of the absolute values of the elements of each row $\|A\|_\infty = \max_{1\le i\le n} \sum_{j=1}^{n} |a_{ij}|$

- Ill conditioned linear equation

$$[H][S] = [R]$$

$$\begin{bmatrix} 1 & 2 \\ 2 & 3.999 \end{bmatrix} \begin{bmatrix} 4 \\ 7.999 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

norm H	$\|H\|_\infty$=	5.999
norm s	$\|R\|_\infty$=	7.999
norm r	$\|S\|_\infty$=	2

$$\begin{bmatrix} 1 & 2 \\ 2 & 3.999 \end{bmatrix} \begin{bmatrix} 4.001 \\ 7.998 \end{bmatrix} = \begin{bmatrix} -3.999 \\ 4 \end{bmatrix}$$

norm H	$\|H'\|_\infty$	5.999
norm s	$\|R'\|_\infty$	7.998
norm r	$\|S'\|_\infty$	4

$\|\Delta R\|_\infty =$	0.001	$\frac{\|\Delta R\|_\infty}{R_\infty} =$	0.000125	$\frac{\|\Delta S\|_\infty}{\|S\|_\infty} =$	2.9995
$\|\Delta S\|_\infty$=	5.999				

$\frac{\|\Delta X\|_\infty / \|X\|_\infty}{\|\Delta C\|_\infty / \|C\|_\infty} =$	23993	43.8	dB
		>10	dB

- Well conditioned linear equation

$$[H][S] = [R]$$

$$\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 7 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

norm H	$\|H\|_\infty$=	5
norm s	$\|R\|_\infty$=	7
norm r	$\|X\|_\infty$=	2

$$\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 4.001 \\ 7.001 \end{bmatrix} = \begin{bmatrix} 1.999 \\ 1.001 \end{bmatrix}$$

norm H	$\|H'\|_\infty$=	5
norm s	$\|R'\|_\infty$=	7.001
norm r	$\|S'\|_\infty$=	1.999

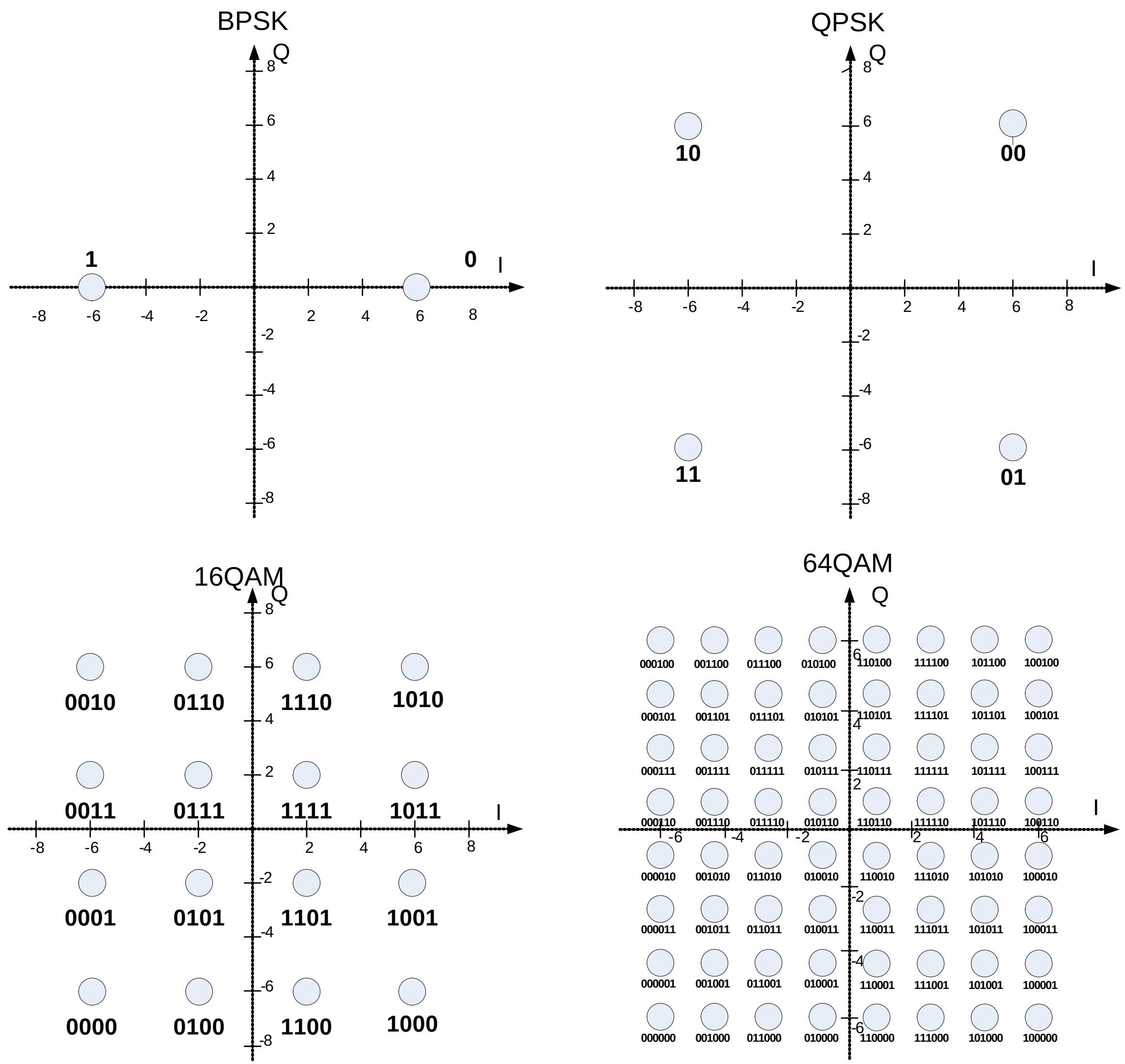
$\|\Delta R\|_\infty =$	0.001	$\frac{\|\Delta R\|_\infty}{R_\infty} =$	0.0001435	$\frac{\|\Delta S\|_\infty}{\|S\|_\infty} =$	0.0005
$\|\Delta S\|_\infty$=	0.001				

$\frac{\|\Delta X\|_\infty / \|X\|_\infty}{\|\Delta C\|_\infty / \|C\|_\infty} =$	3.5	5.4	dB
		<10	dB

MIMO Considerations

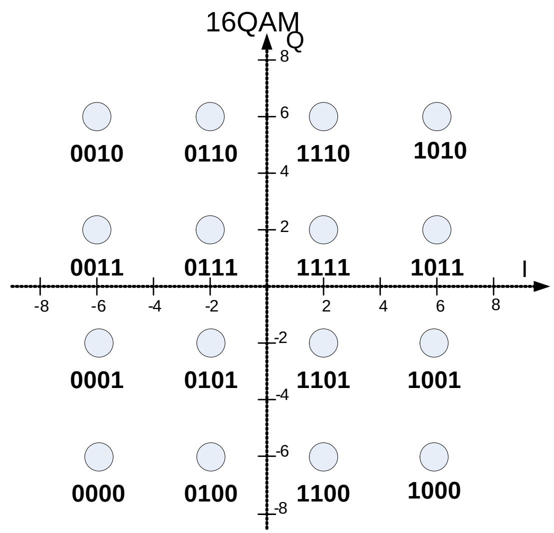
Constellations

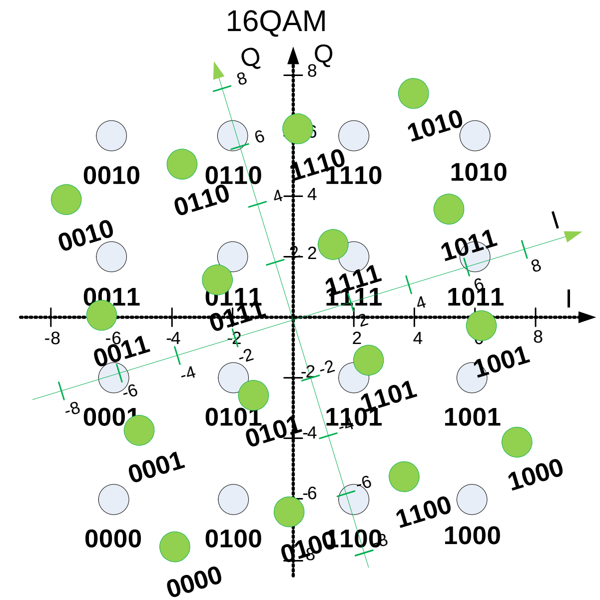
- In WiMAX and LTE one antenna transmits one of the constellation states per symbol
- In Transmit Diversity configuration the second antenna transmits the same information, but it may arrive with a different phase at the receiver
  - The difference between the two paths can be adjusted at the transmitter and the signals will reinforce each other

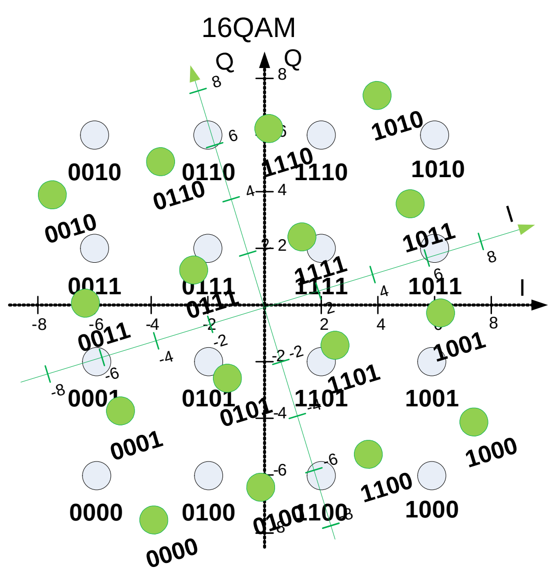


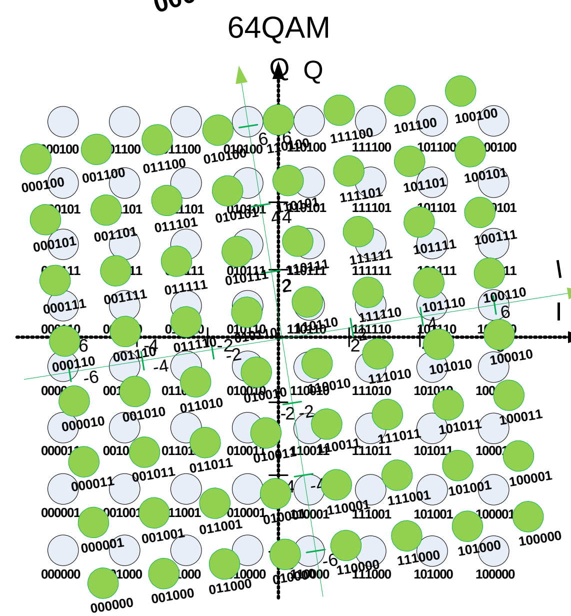
Shifted Constellations

- In Spatial Multiplexing the antennas transmit different data, so they would interfere with each other
- Ideally both constellations should be rotated, so they occupy a different space in the constellation
- This allows to detect each antenna transmission, but the tolerable SNR is largely diminished









MIMO Configurations

- The Maximum Likelihood Detector (MLD) has to consider possible combinations of s_0 and $s_{1,}$ which could be a large number
- For two symbols the total number of combinations for 64QAM, 16QAM and QPSK is 4,368
- This number can be reduced to 1,092 combinations if a quadrant approach is used.
  - In this approach, instead of checking all possible combinations, quadrants are tested first, eliminating the rejected quadrant combinations

$D^2(s_0, s_1) = \{\lvert r_0 - h_{00}s_0 - \mathbf{h_{10}}\mathbf{s_1}\rvert^2 + \lvert r_1 - \mathbf{h_{01}}s_0 - h_{11}s_1\rvert^2\}$	**Maximum Likelihood Detector**

- The table below shows the best combinations for the various numbers of antennas

	Number of RX antennas			
Number of TX antennas	1	2	3	4
1	Baseline	Downlink: MRC Uplink: Collaborative MIMO	MRC	MRC
2	STC (Matrix A)	2xSMX (Matrix B) STC+ 2xMRC (Matrix A)	2xSMX (Matrix B) STC+3xMRC (Matrix A)	STC+ 4xMRC (Matrix A)
4	STC (Matrix A)	2xSMX (Matrix B) STC+ 2xMRC (Matrix A)	2xSMX (Matrix B) STC+3xMRC (Matrix A)	4xSMX (Matrix C)

Adaptive MIMO Switching (AMS)

- Transmit Diversity presents a higher throughput than Spatial Multiplexing at low SNIR levels, whereas Spatial Multiplexing results in a higher throughput than Transmit diversity for high SNIR levels
- AMS automatically chooses the solution that gives the best throughput at each location

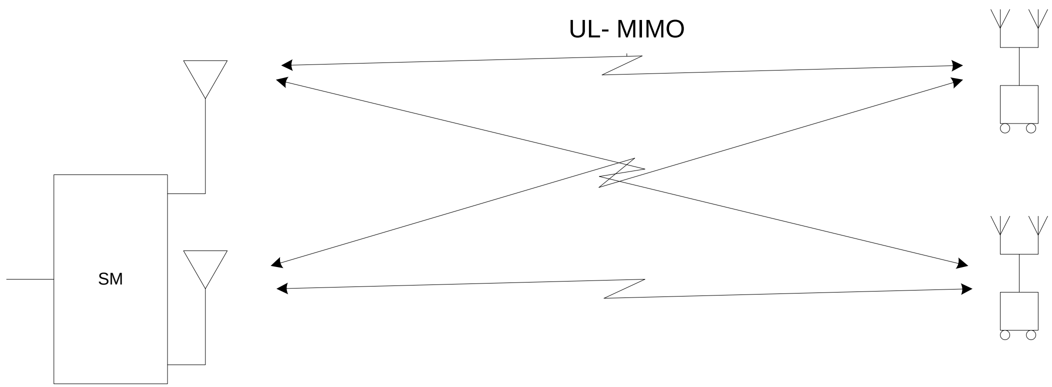
5.5 Multi-user MIMO

Uplink MIMO

Multi-user MIMO

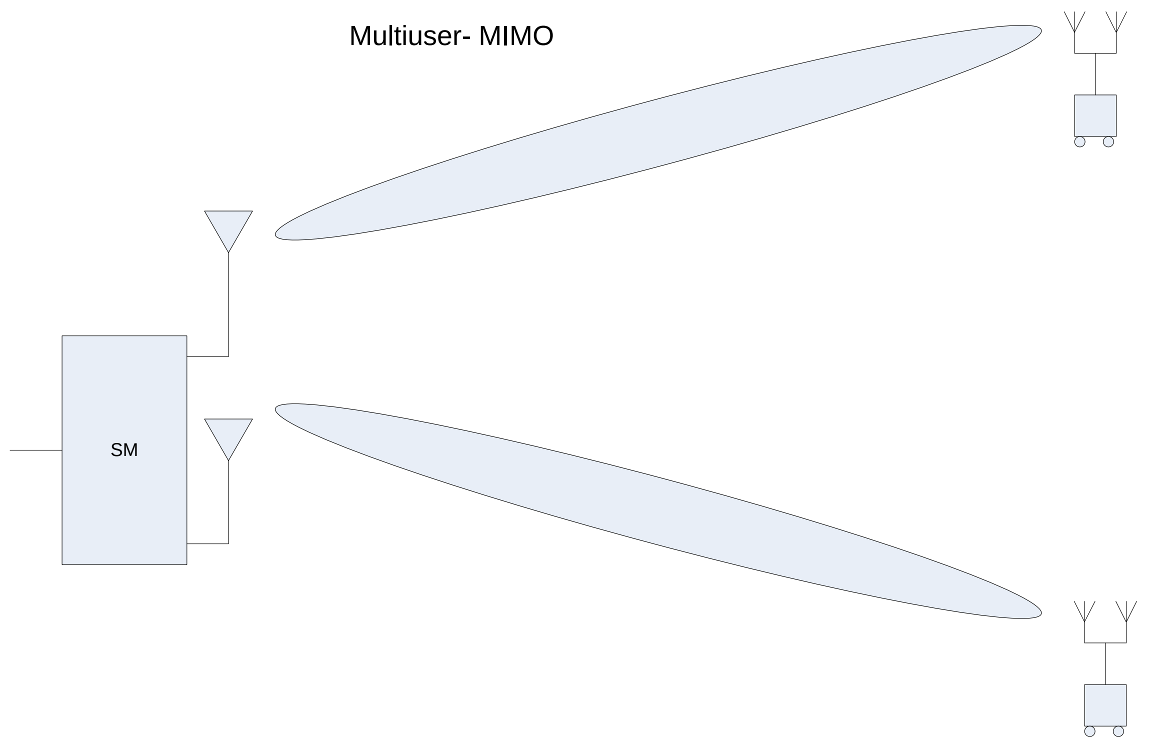
Uplink MIMO (UL-MIMO)

- This is also a Spatial Multiplexing (SM) technique but used in the upstream only
- It is also known as Uplink Collaborative MIMO
- Because transmissions arrive from different locations they are non coherent between themselves, thus providing a better performance than the one obtained in the downlink spatial multiplexing, although the self interference issues continue to be present



Multi-User MIMO

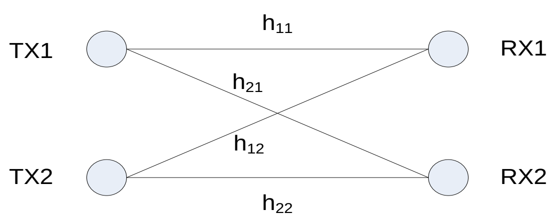
- In Multi-User MIMO the information transmitted by each antenna is sent to different users
- This technique uses beam-forming to reduce interference between users



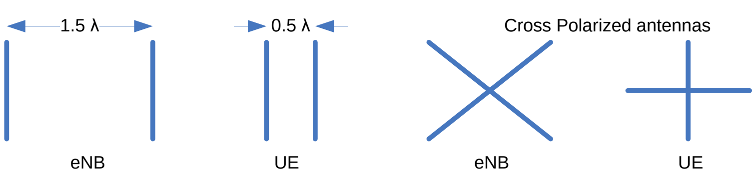
6. Antenna Correlation

Antenna Correlation

- Multiple antenna systems are said to have correlated antennas if the signals received by the receive antennas are coherent, i.e. are similar to each other
- When the received signals are differentiated (non-coherent) the antennas are said to be uncorrelated
- Channel (H) between two transmit and receive antennas is represented by four virtual connections (h_{nn}) for each delayed path
  - The matrix that defines the virtual connections of each delayed complex channel path is given below
- The correlation between antennas is a function of the local scattering and is a function of the Angular Spread (AS), Angle of Arrival (AoA) and Direction of Travel (DoT)
  - This correlation is not constant, varying significantly over a geographical area
- The correlation between antennas can be expressed in terms of signal (ρ) observed at each antenna element
- ITU Advanced Antenna Models replace R_{tx} by R_{eNB}, ρ_{tx} by α and ρ_{rx} by β
- The α and β values represent the different channel types and range from 0 to 1



	Low correlation	Medium correlation	High correlation
α	0	0.3	0.9
β	0	0.9	0.9



High Correlation

Medium Correlation

$\boldsymbol{H} = \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix}$	**Matrix H of a complex channel path**
$\boldsymbol{r} = \begin{vmatrix} r_1 \\ r_2 \end{vmatrix} = \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix} \begin{vmatrix} x_1 \\ x_2 \end{vmatrix} + \begin{vmatrix} n_1 \\ n_2 \end{vmatrix}$	**Output complex signal for a single delayed path**
$\rho = \dfrac{E(h_{11}h_{12}^{\ *})}{\sqrt{E(h_{11}h_{11}^{\ *})}\sqrt{E(h_{12}h_{12}^{\ *})}}$	**Output complex signal for a single delayed path**
$E(h_{11}h_{11}^{\ *}) = 1$	**Expectancy normalization**
$\rho_{tx} = E(h_{11}h_{12}^{\ *})$	**Correlation between transmit antennas as measured at antenna 1**
$\rho_{rx} = E(h_{22}h_{12}^{\ *})$	**Correlation between receive antennas as measured at antenna 2**
$R = R_{tx} \otimes R_{rx}$	**Correlation matrix**
$R_{tx} = \begin{bmatrix} 1 & \rho_{tx} \\ \rho_{tx}^{*} & 1 \end{bmatrix}$	**Transmit correlation matrix**
$R_{rx} = \begin{bmatrix} 1 & \rho_{rx} \\ \rho_{rx}^{*} & 1 \end{bmatrix}$	**Receive correlation matrix**
$R_{eNB} = \begin{pmatrix} 1 & \alpha \\ \alpha^{*} & 1 \end{pmatrix} \quad R_{UE} = \begin{pmatrix} 1 & \beta \\ \beta^{*} & 1 \end{pmatrix}$	**eNB and UE Antenna correlation**
$R_{spatial} = R_{eNB} \otimes R_{UE} = \begin{bmatrix} 1 & \propto \\ \propto^{*} & 1 \end{bmatrix} \otimes \begin{bmatrix} 1 & \beta \\ \beta^{*} & 1 \end{bmatrix} = \begin{bmatrix} 1 & \beta & \alpha & \alpha\beta \\ \beta^{*} & 1 & \alpha\beta^{*} & \alpha \\ \alpha^{*} & \alpha^{*}\beta & 1 & \beta \\ \alpha^{*}\beta^{*} & \alpha^{*} & \beta^{*} & 1 \end{bmatrix}$	**Spatial antenna correlation**

Correlation

- The correlation coefficient, denoted by *r* tells us how closely data in a scatterplot fall along a straight line
- The closer that the absolute value of *r* is to one, the closer that the data are described by a linear equation
- Data with values of *r* close to zero show little to no straight-line relationship

- The data we are working with are paired data, each pair of which will be denoted by (x_i, y_i)
  - Calculate $\bar{x}$, the mean of all of the first coordinates of the data x_i
  - Calculate $\bar{y}$, the mean of all of the second coordinates of the data y_i
  - Calculate s_x the sample standard deviation of all of the first coordinates of the data x_i
  - Calculate s_y the sample standard deviation of all of the second coordinates of the data y_i
- Use the formula $(z_x)_i = (x_i - \bar{x}) / s_x$ and calculate a standardized value for each x_i
- Use the formula $(z_y)_i = (y_i - \bar{y}) / s_y$ and calculate a standardized value for each y_i
- Multiply corresponding standardized values: $(z_x)_i(z_y)_i$
- Add the products from the last step together
- Divide the sum from the previous step by $n - 1$, where n is the total number of points in our set of paired data
  - The result of all of this is the correlation coefficient *r*

Correlation

- We begin with a listing of paired data: (1, 1), (2, 3), (4, 5), (5,7)
  - The mean of the *x* values, the mean of 1, 2, 4, and 5 is $\bar{x} = 3$
  - We also have that $\bar{y} = 4$
  - The standard deviation of the *x* values is $s_x = 1.83$ and $s_y = 2.58$
- The table below summarizes the other calculations needed for *r*
  - The sum of the products in the rightmost column is 2.969848
  - Since there are a total of four points and 4 – 1 = 3, we divide the sum of the products by 3
- This gives us a correlation coefficient of $r = 2.969848/3 = 0.989949$

x	y	z_x	z_y	$z_x z_y$
1	1	-1.09544503	-1.161894958	1.272792057
2	3	-0.547722515	-0.387298319	0.212132009
4	5	0.547722515	0.387298319	0.212132009
5	7	1.09544503	1.161894958	1.272792057

Pearson product-moment correlation coefficient

- In statistics, the **Pearson product-moment correlation coefficient** (sometimes referred to as the **PPMCC** or **PCC** or **Pearson's *r***) is a measure of the linear correlation (dependence) between two variables *X* and *Y*, giving a value between +1 and −1 inclusive, where 1 is total positive correlation, 0 is no correlation, and −1 is total negative correlation
  - It is widely used in the sciences as a measure of the degree of linear dependence between two variables

$$r = \frac{\sum_{i=1}^{n}(X_i - \bar{X})(Y_i - \bar{Y})}{\sqrt{\sum_{i=1}^{n}(X_i - \bar{X})^2} * \sqrt{\sum_{i=1}^{n}(Y_i - \bar{Y})^2}}$$

- The correlation coefficient ranges from −1 to 1
- A value of 1 implies that a linear equation describes the relationship between *X* and *Y* perfectly, with all data points lying on a line for which *Y* increases as *X* increases
- A value of −1 implies that all data points lie on a line for which *Y* decreases as *X* increases
- A value of 0 implies that there is no linear correlation between the variables
- More generally, note that $(X_i - X)(Y_i - Y)$ is positive if and only if X_i and Y_i lie on the same side of their respective means
  - Thus the correlation coefficient is positive if X_i and Y_i tend to be simultaneously greater than, or simultaneously less than, their respective means
  - The correlation coefficient is negative if X_i and Y_i tend to lie on opposite sides of their respective means.

Antenna Correlation

- **Variance** is a measure of the variability or spread in a set of data
  - Mathematically, it is the average squared deviation from the mean score
    - $Var\ (X) = \sum_{i=0}^{N} (X_i - X)^2/N = \sum_{i=0}^{N} {x_i^2}/{N}$

      N is the number of scores in a set of scores
      X is the mean of the *N* scores.
      X_i is the *i*th raw score in the set of scores
      x_i is the *i*th deviation score in the set of scores
      Var (*X*) is the variance of all the scores in the set
- **Covariance** is a measure of the extent to which corresponding elements from two sets of ordered data move in the same direction
    - $Var\ (X) = \sum_{i=0}^{N} (X_i - X)(Y_i - Y)/N = \sum_{i=0}^{N} {x_i y_i}/{N}$

      N is the number of scores in each set of data
      X is the mean of the *N* scores in the first data set
      X_i is the *i*th raw score in the first set of scores
      x_i is the *i*th deviation score in the first set of scores
      Y is the mean of the *N* scores in the second data set
      Y_i is the *i*th raw score in the second set of scores
      y_i is the *i*th deviation score in the second set of scores
      Cov (*X*, *Y*) is the covariance of corresponding scores in the two sets of data

Antenna Correlation

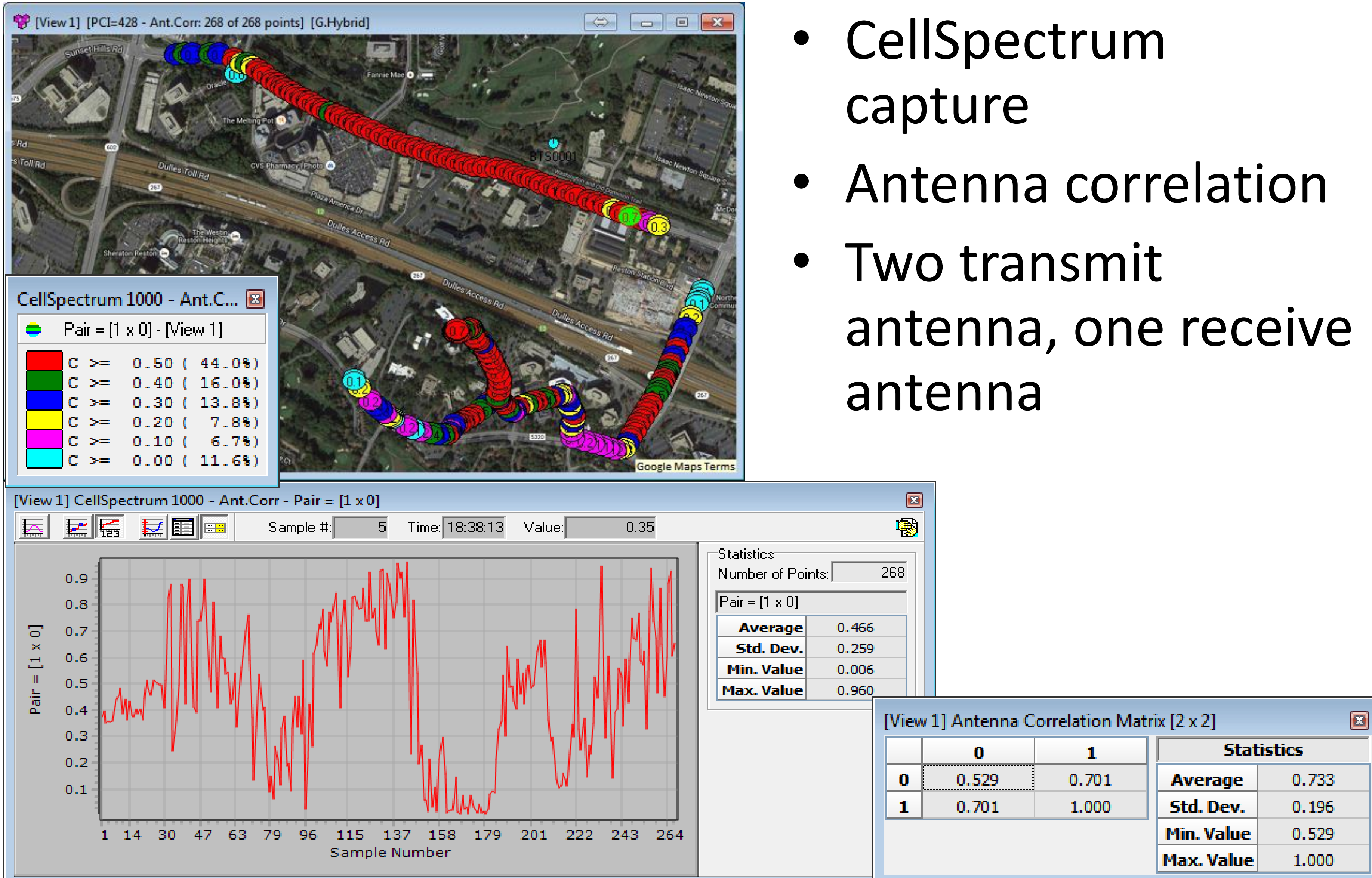
- Variance and covariance are often displayed together in a variance-covariance matrix, (aka, a covariance matrix)
  - Variances appear along the diagonal and covariance appears in the off-diagonal elements, as shown below
    - $\boldsymbol{V} = \begin{bmatrix} \sum_{i=0}^{N} x_i^2/N & \sum_{i=0}^{N} x_i y_i/N \\ \sum_{i=0}^{N} y_i x/N & \sum_{i=0}^{N} y_i^2/N \end{bmatrix}$

V is a *c* x *c* variance-covariance matrix
N is the number of scores in each of the *c* data sets
x_i is a deviation score from the *i*th data set
$\Sigma\, x_i^2$ / N is the variance of elements from the *i*th data set
$\Sigma\, x_i\, x_j$ / N is the covariance for elements from the *i*th and *j*th data sets

Correlation Matrix

- **Correlation Matrix** is a quantity closely related to the covariance matrix is the correlation matrix, the matrix of Pearson product-moment correlation coefficients between each of the random variables in the random vector , which can be written
- $corr(X) = (diag(\Sigma))^{\frac{-1}{2}}\,\Sigma(diag(\Sigma))^{\frac{-1}{2}}$
  - $diag(\Sigma)$ is the matrix of the diagonal elements of Σ (i.e., a diagonal matrix of the variances of X_i for i=1,2,…n
- Equivalently, the correlation matrix can be seen as the covariance matrix of the standardized random variables $X_i/\sigma(X_i)$ for i=1,2,…n

Antenna Correlation



- CellSpectrum capture
- Antenna correlation
- Two transmit antenna, one receive antenna

7. AAS

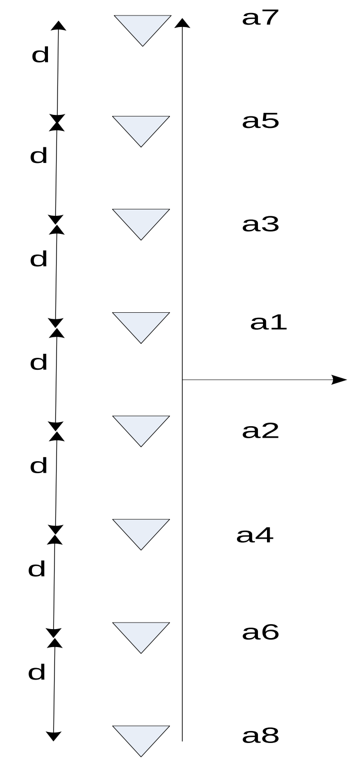
Antenna Array System (AAS)
Advanced Antenna System (AAS)
Adaptive Antenna Steering (AAS)
Beamforming

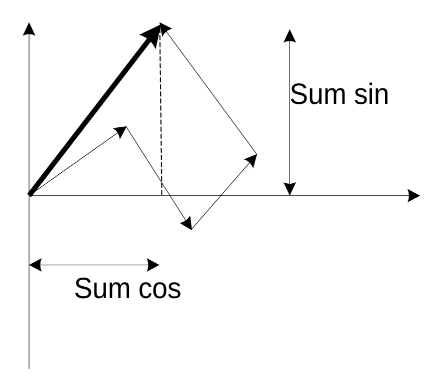
Antenna Array System (AAS) Advanced Antenna System (AAS) or Adaptive Antenna Steering (AAS) or Beamforming

- Advanced antenna systems can be built by multiple elements which are fed with different signal phases and can generate nulls and poles at certain directions
- This feature is used to reinforce signals and cancel interferences
- Two of the main methods, direction of arrival and antenna steering, are described next
- Direction of Arrival (DoA) beamforming is done by detecting the direction with which the signal and interferers arrive, reinforcing the first one and canceling the others
- The maximum number of cancelled signal is equal to the number of antenna elements minus one
- The re-enforcement is usually of the order of few dB and the canceling is not complete either
- As different implementations have large variations, these parameters have to be specified at design time based on the equipment used

Antenna Steering or Beamforming

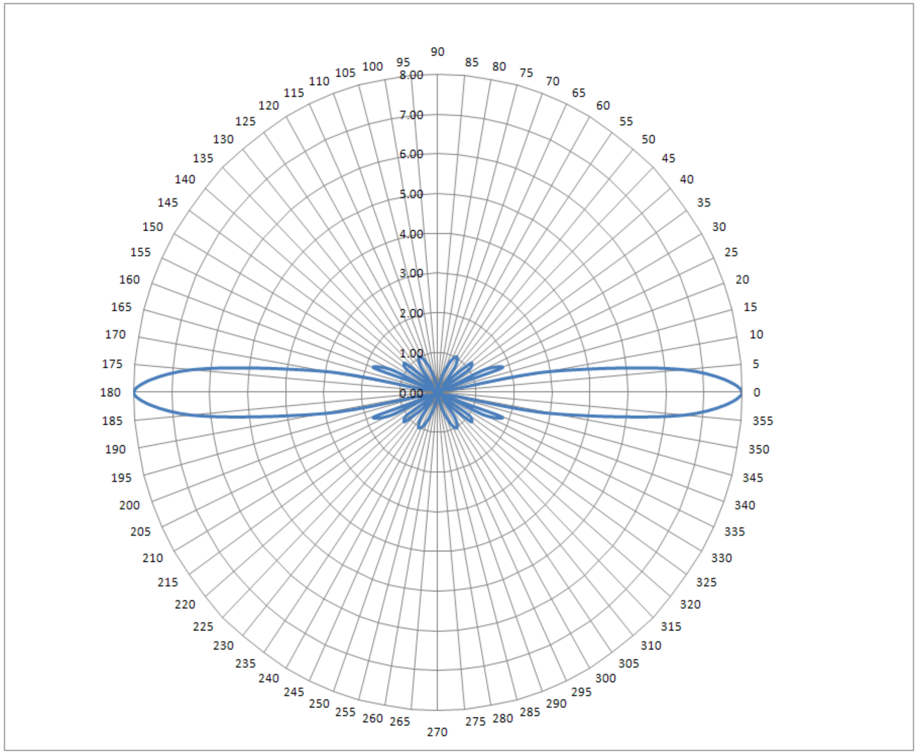
- It is used to direct the signal transmission or reception towards the desired signal or away from interferers
- The concept is based on the combination of signals from an array of antennas
- These arrays are also known as smart antennas and the figure illustrates an array (linear) of eight antennas, spaced by a distance d
- Although the array is made of omni antennas it has a directional pattern that can be calculated by adding the signal received from each antenna at a certain distance

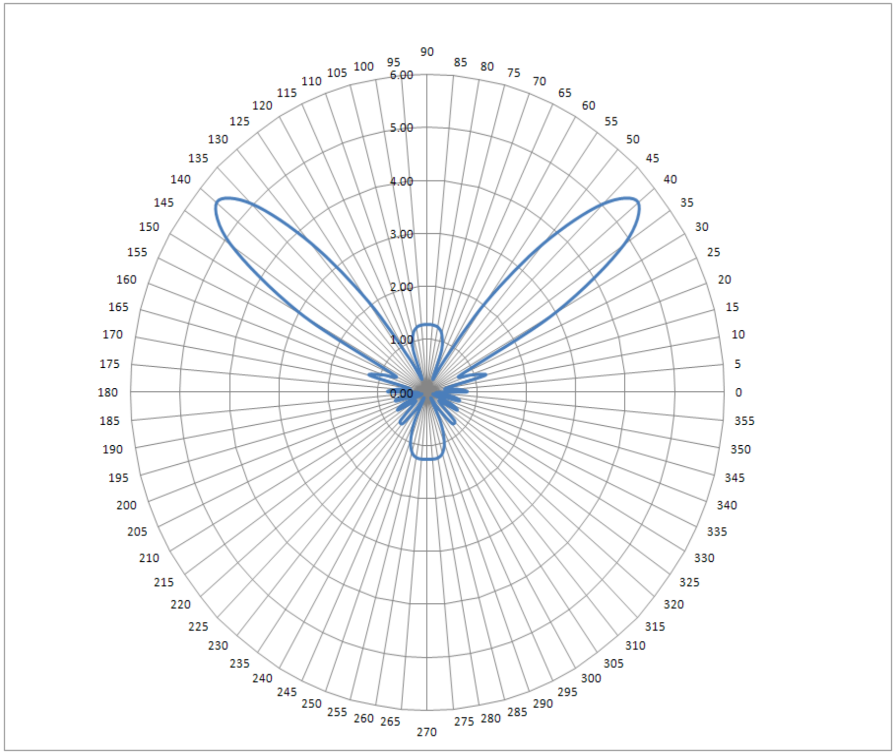




Antenna Steering or Beamforming

- This combination results in the pattern of figure on the left for eight antennas separated by λ/2
- This pattern can be modified by changing the phases of the signals to each antenna, as shown in figure to the right





Antenna Steering or Beamforming

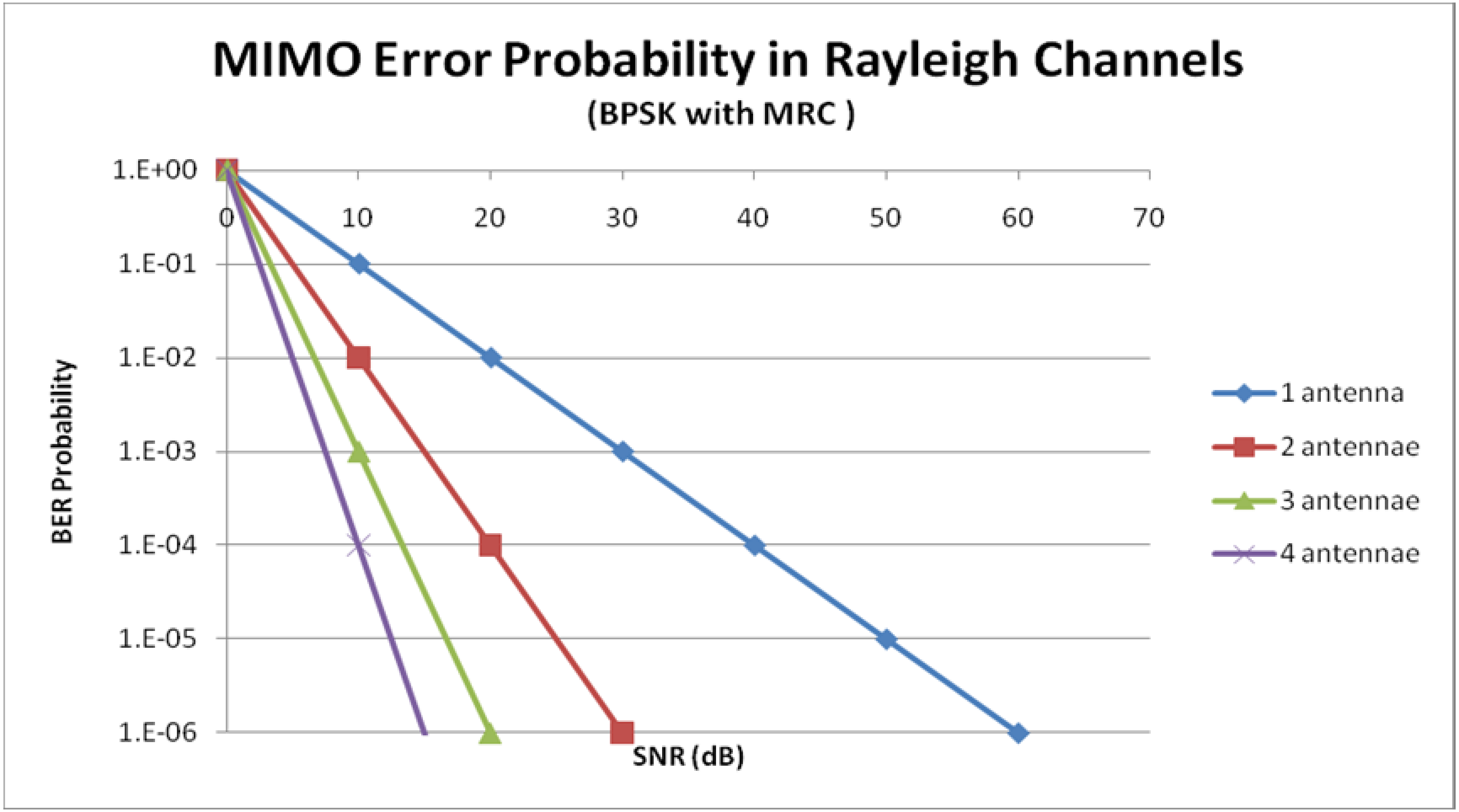
- The adaptive beamforming uses the information of symbols received by the array, to define the desired pattern
  - The signal phases to different antennas are adjusted dynamically
- The array antennas can be distributed in a line, forming linear arrays, or on a plane, forming planar arrays
- The number of elements in the array defines how many directions can be chosen simultaneously
  - This number is equal to N-1 directions, where N is the number of elements in the array
  - The phases can be adjusted to enhance the reception for the direction or cancel the signal from it

8. MIMO Performance

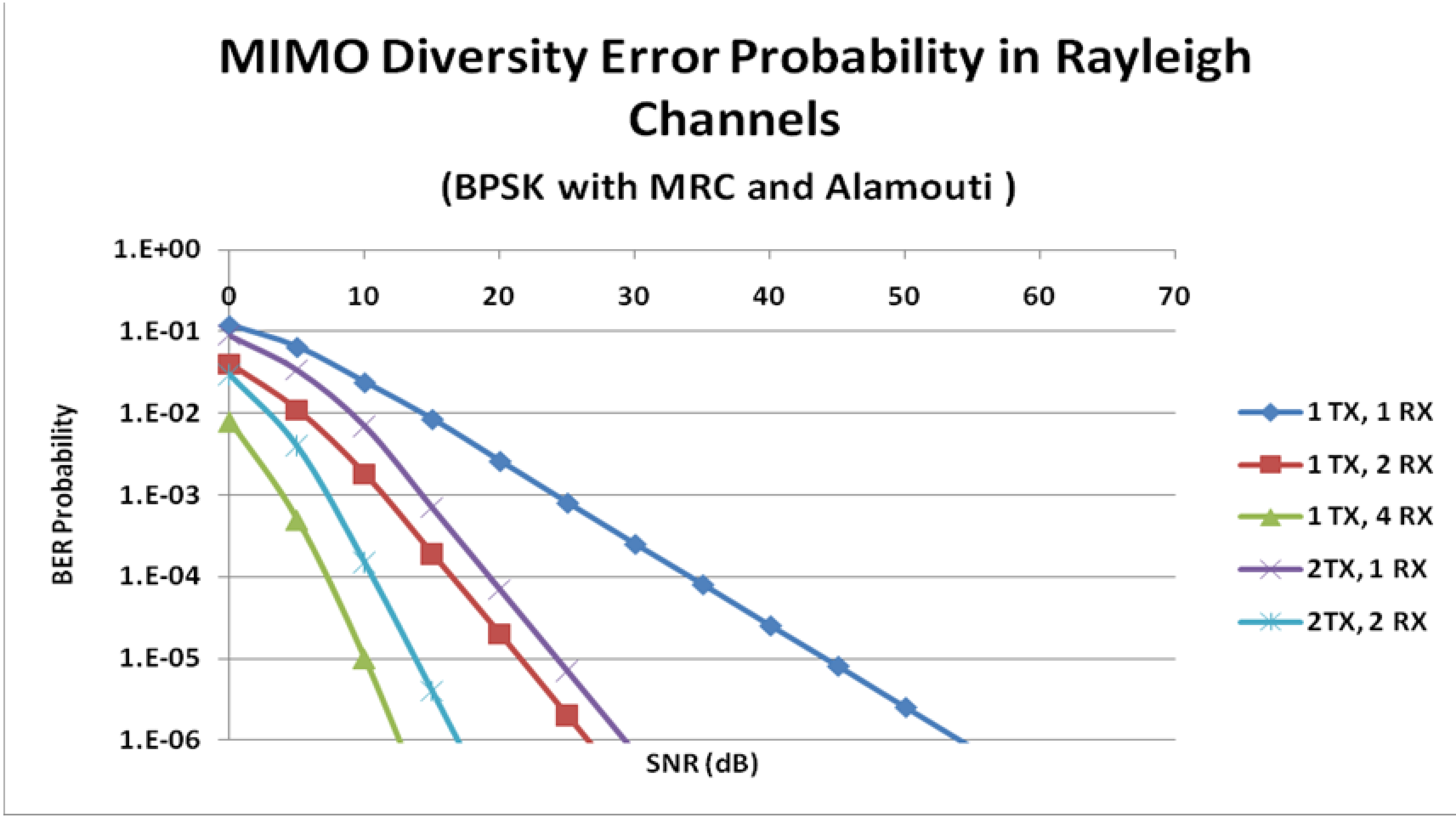
MIMO Performance

- The performance of the different MIMO methods is derived from literature and we display here average values
- The figures show the SNR required for different levels of Bit Error Rate depending on the number of antennas and modulation scheme

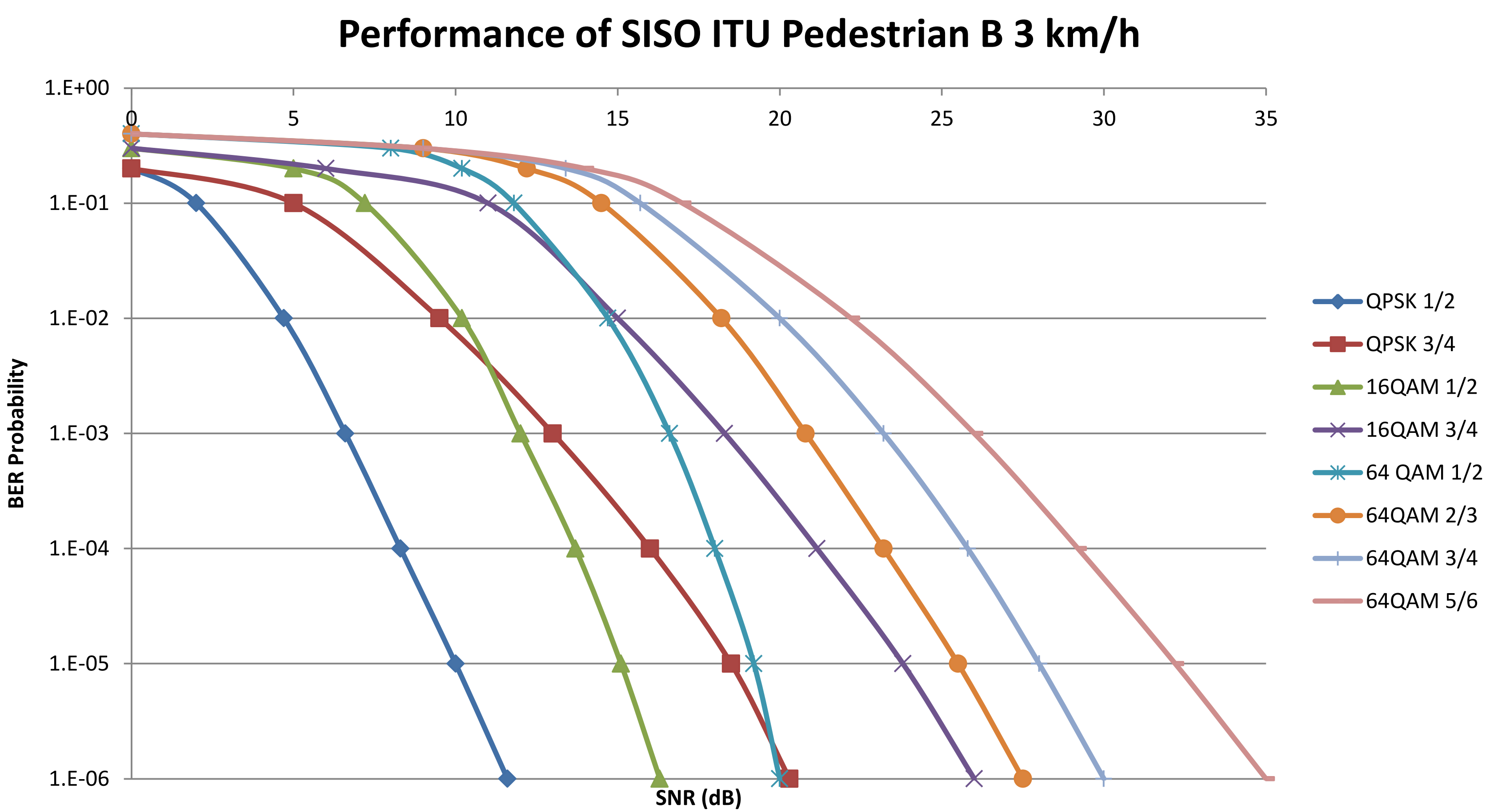
MIMO Error Probability in a Rayleigh Channel



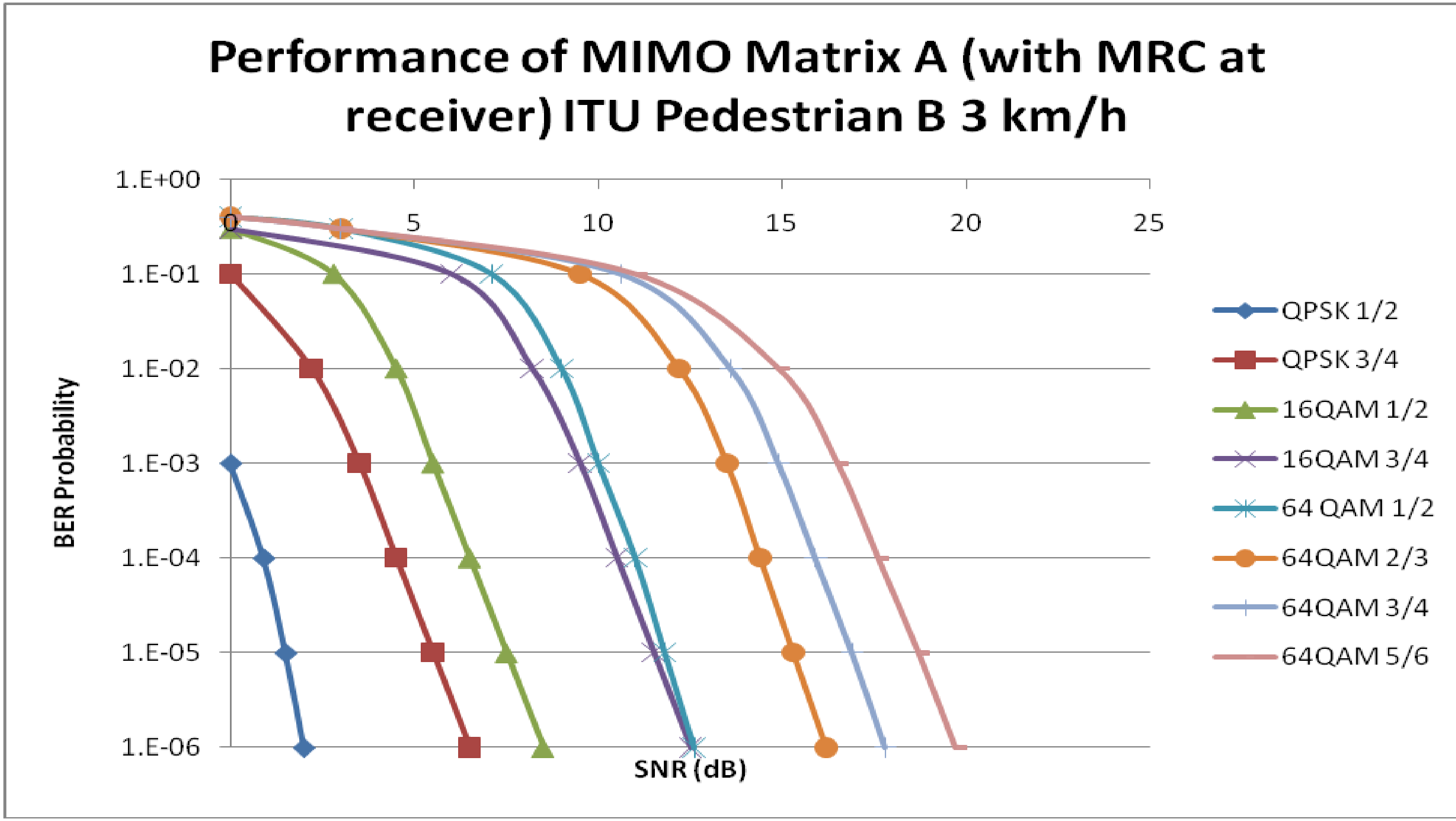
MIMO Diversity Error Probability in a Raleigh Channels



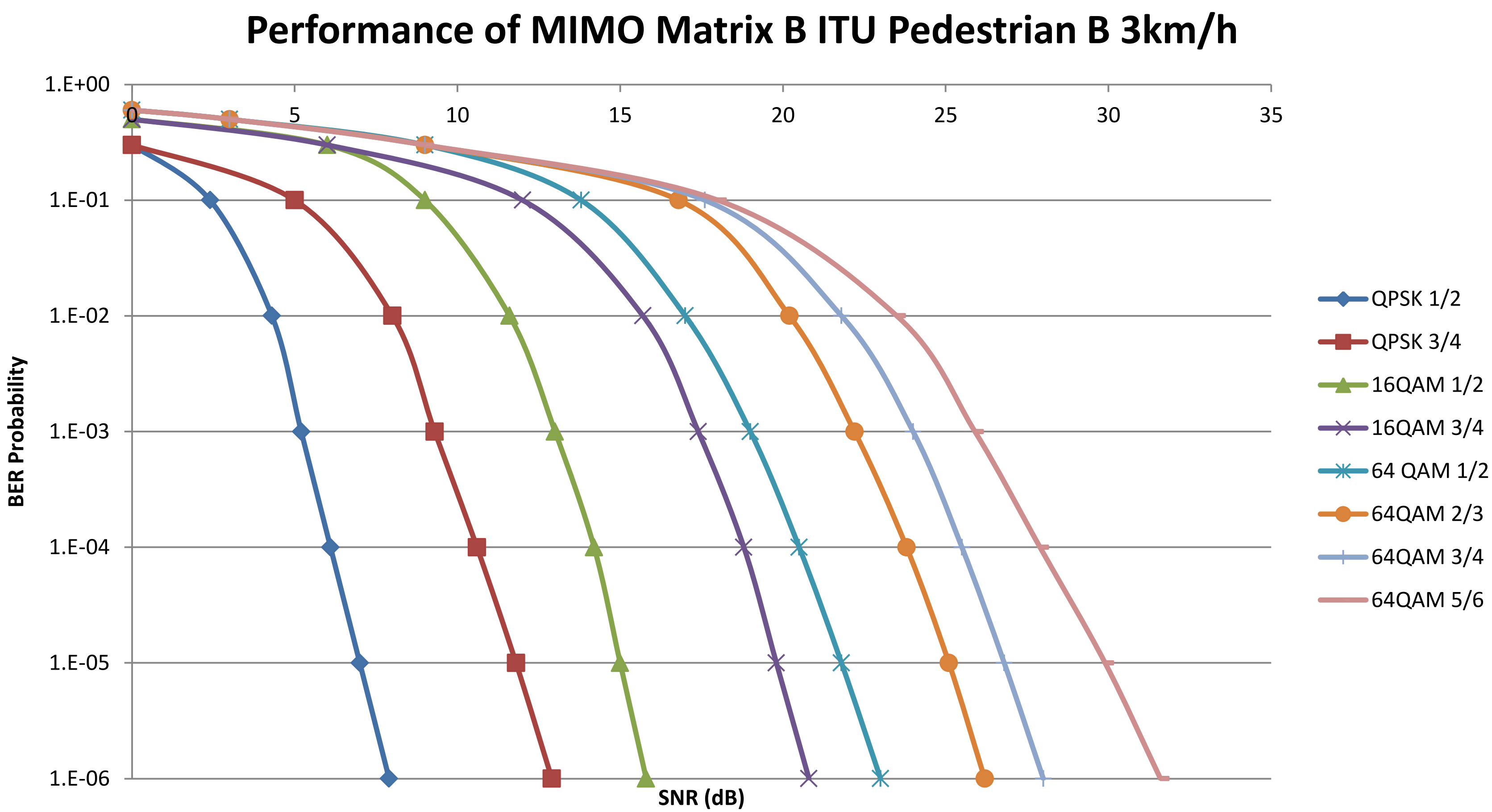
Performance of SISO ITU for Pedestrian B



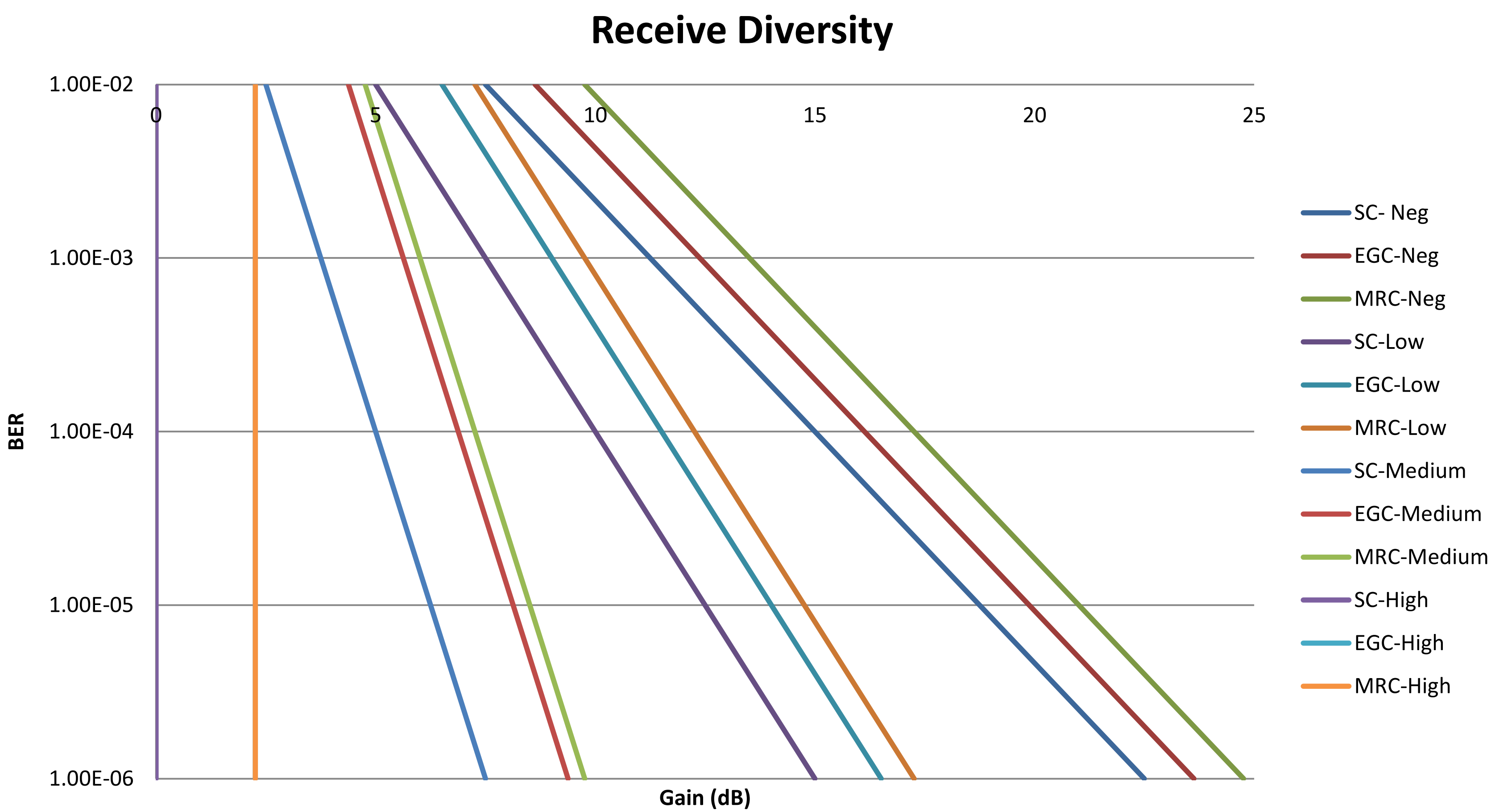
Performance of MIMO Matrix A



Performance of MIMO Matrix B

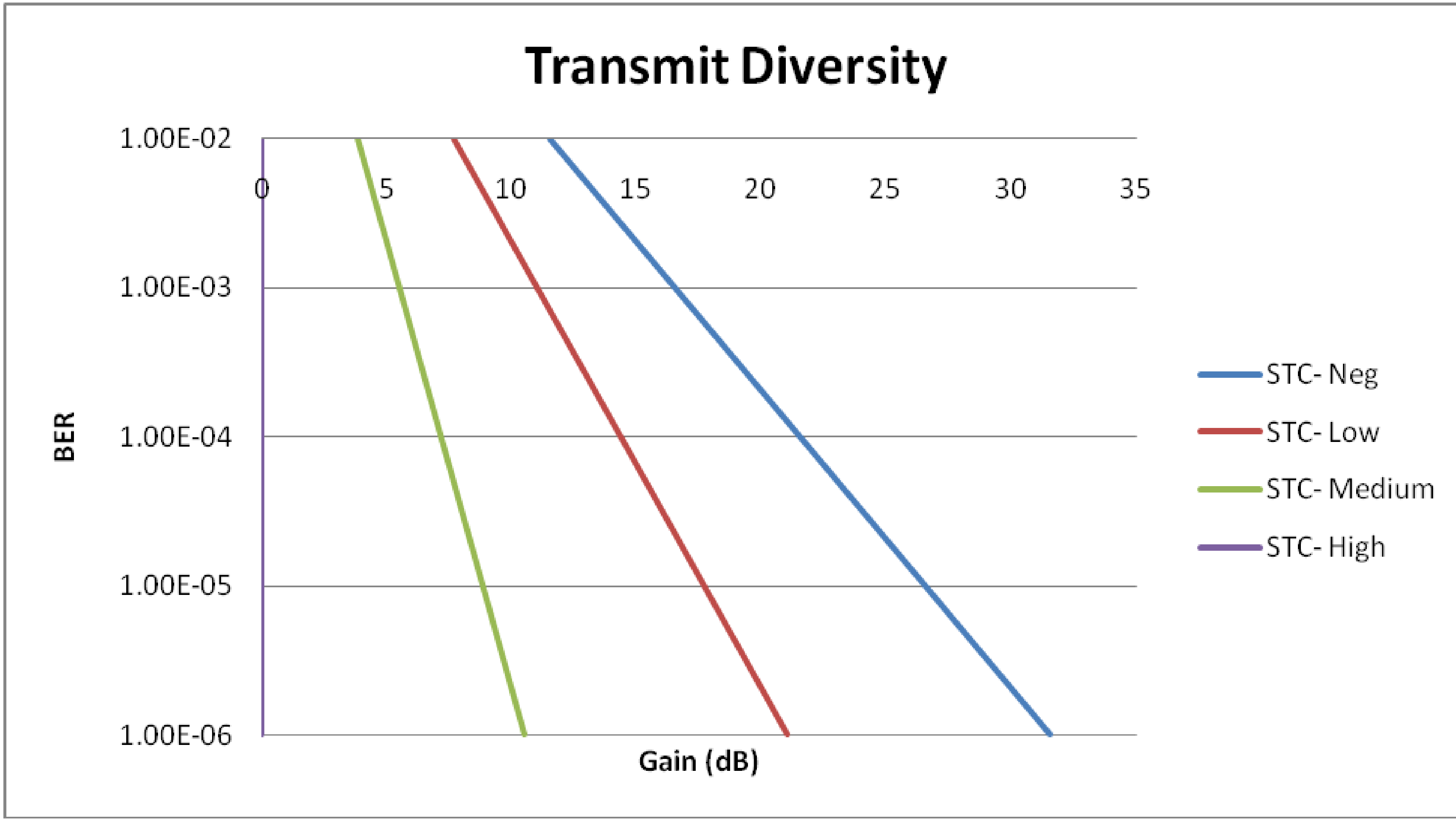


Performance of Receive Diversity Technique



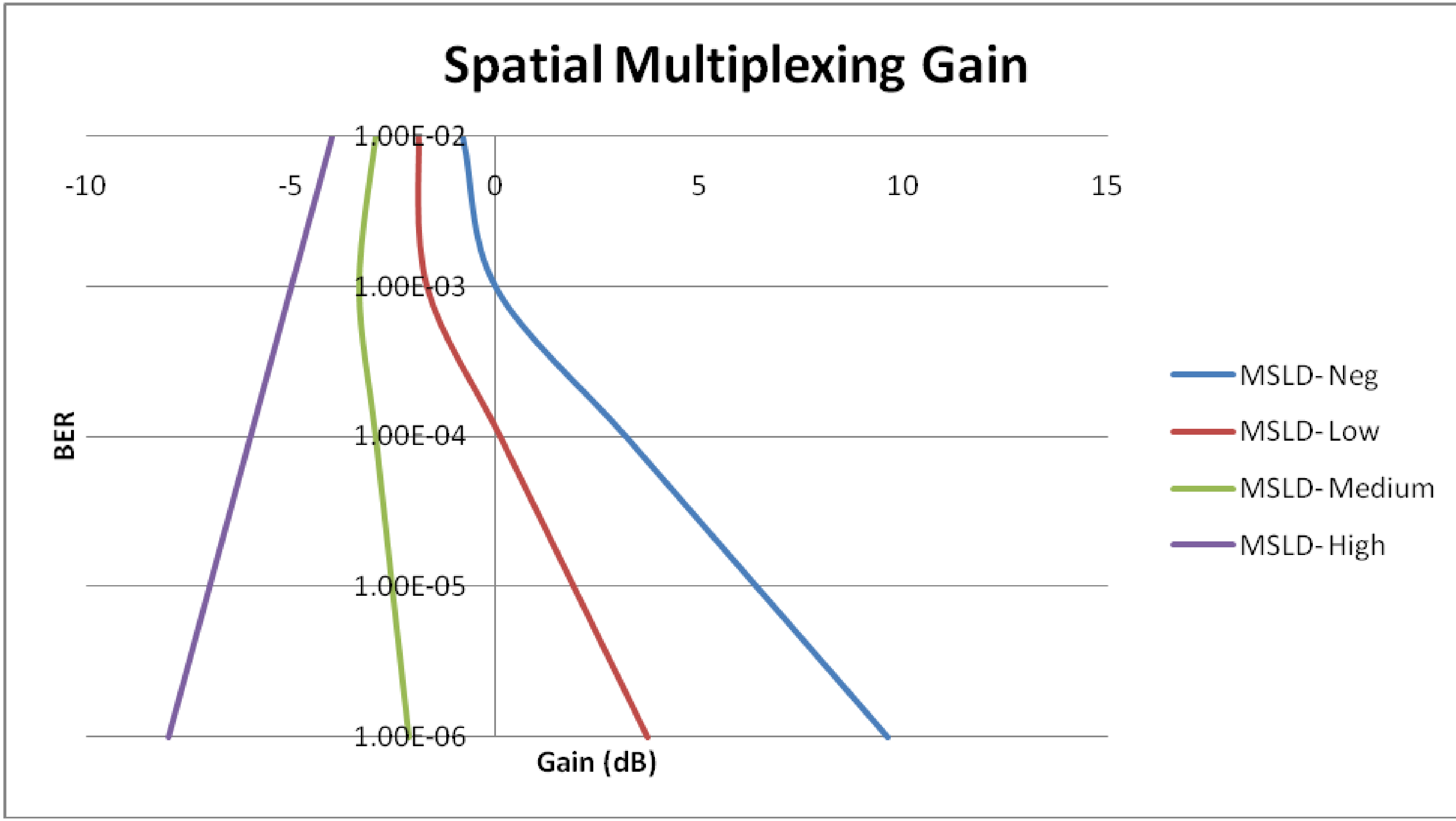
Gain figures are theoretical average values that should be considered carefully
Gains considerations should be made to compensate for losses

Performance of Transmit Diversity Technique



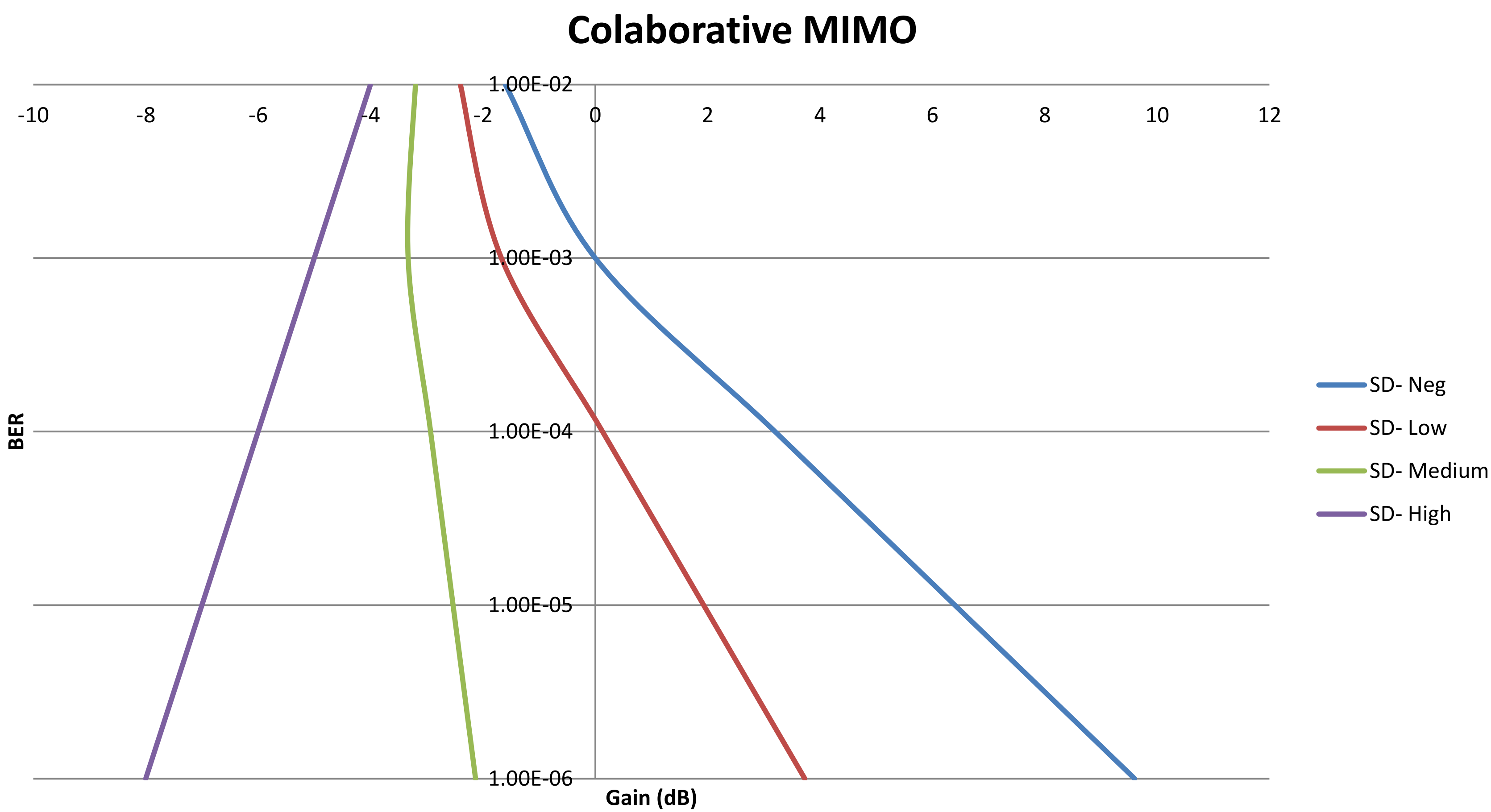
Gain figures are theoretical average values that should be considered carefully
Gains considerations should be made to compensate for losses

Performance of Spatial Multiplexing Gain



Gain figures are theoretical average values that should be considered carefully
Gains considerations should be made to compensate for losses

Performance of Collaborative MIMO



Gain figures are theoretical average values that should be considered carefully
Gains considerations should be made to compensate for losses

9. CelPlan New Products

CellSpectrum

CellDesigner

9.1 CellDesigner

A new Generation of Planning Tools

A collaborative work with operators

Your input is valuable

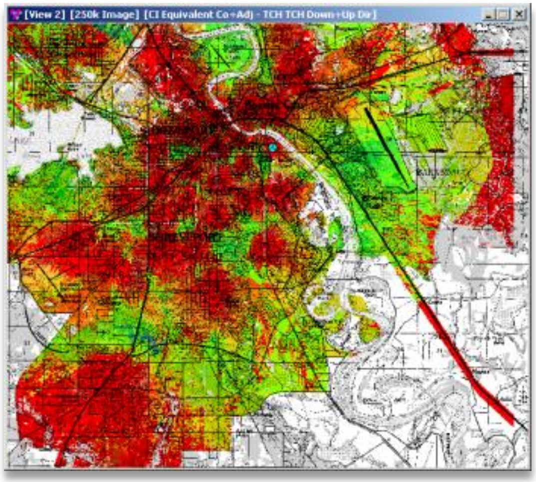
CellDesigner

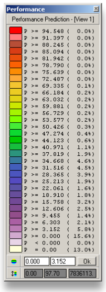
- CellDesigner is the new generation of Planning and Optimization tools
- Wireless networks became so complex that it requires a new generation of tools, capable of:
  - Documenting the physical deployments
  - Documenting network parameters for each technology
  - Flexible data traffic modelling (new services, new UE types)
  - Traffic allocation to different technologies
  - Fractional Resouce Planning
  - Performance evaluation
  - Integrated backhaul

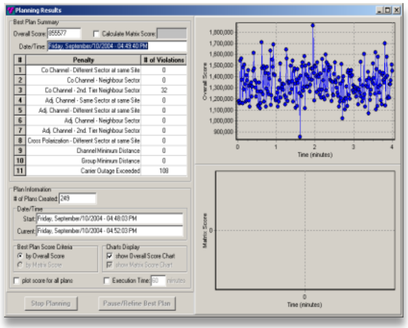
CellDesigner™

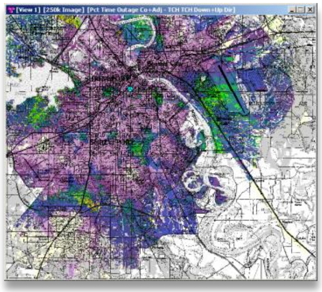
Simultaneous Multi-Technology Support

- Supports all wireless technology standards:
  - LTE–A (TDD and FDD), WiMAX, WI-FI, WCDMA (UMTS), HSPA, HSPA+, IS2000 (1xRTT, EVDO), GSM (including Frequency Hoping), GPRS, EDGE, EDGE-E, CDMA One, PMR/LMR (Tetra and P25), MMDS/LMDS, DVB-T/H, and Wireless Backhaul
- Full network representation
  - Site, Tower, Antenna Housing, Antenna System, Sector, Cell, Radio
  - Full network parameter integration
  - KPI integration
- Full implementation of the Korowajczuk 3D model, capable of performing simultaneously outdoor and indoor multi-floor predictions
- Multi-technology dynamic traffic simulation









CellDesigner™

Automatic Resource Planning (ARP)

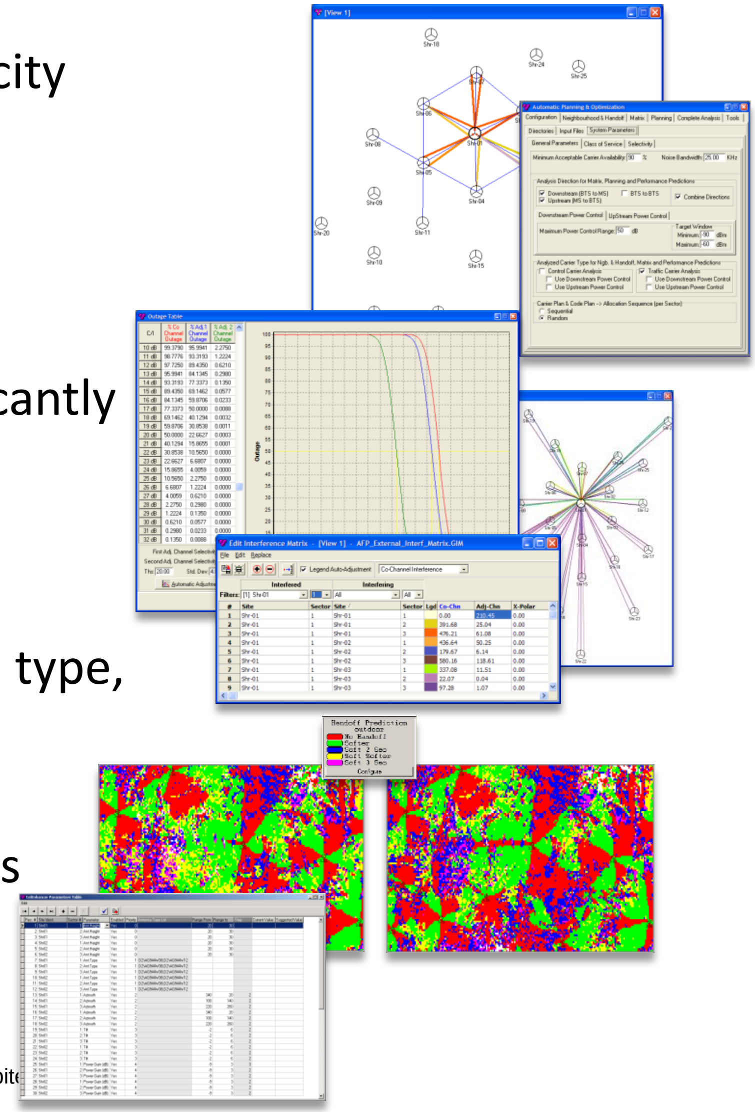
- Enables the dramatic increase of network capacity and performance
- Handover, Frequency and Code Optimization
- Automatically and efficiently optimizes handoff thresholds, neighbor lists, and frequency plans
- Patent-pending methodology capable of significantly increasing cell capacity (SON & ICIC)

Automatic Cell Planning (ACP)

- Footprint and interference enhancement
- Allows optimization of radiated power, antenna type, tilt, azimuth, and height

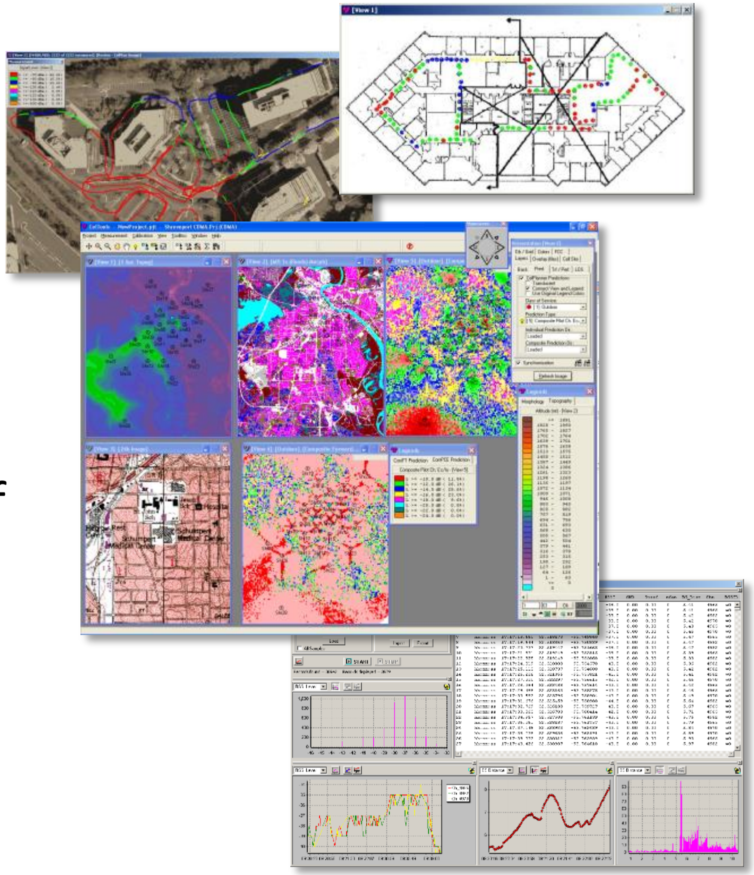
Performance Predictions

- Overall performance prediction per service class (bearer)



CelPlan Wireless Solutions & Consulting

CellDesigner™



Google Earth Integration

- Capable of presenting predictions and measurements live in Google Earth’s 3D environment

Network Master Plan (NMP)

- Patent-pending methodology that simplifies SON and ICIC

Integration of Field Measurement Data

- Collection of data from virtually any type of measurement equipment and any format
- Automatic extraction of propagation parameters

Integration of KPIs

- Comparison reports between reported and calculated KPIS

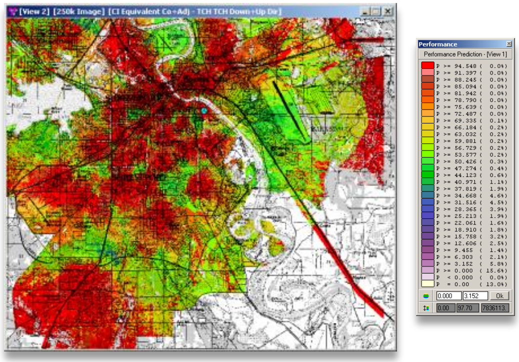
CellDesigner™

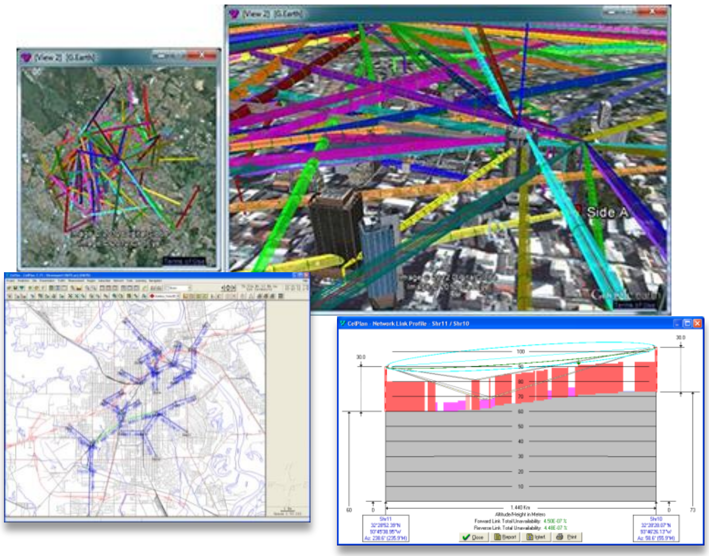
GIS Database Editor

- Allows the editing and processing of geographical databases

Backhaul Planning

- Calculates network interconnections, interference analysis & reporting for point-to-point, microwave transmission links
- Can display obstruction in Fresnel zones as well as the path loss
- Calculates attenuation caused by diffraction.
- Calculates rain attenuation for each link
- Provides link performance and compares against the requirements established by ITU-R





Thank You!

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Questions?